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Peer-review status of attached file:

Peer-reviewed

Citation for published item:

Franco, Sebastián and Uranga, Angel (2006) 'Dynamical SUSY breaking at meta-stable minima from D-branes at obstructed geometries.', *Journal of high energy physics.*, 2006 (06). 031.

Further information on publisher's website:

<http://dx.doi.org/10.1088/1126-6708/2006/06/031>

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Dynamical SUSY Breaking at Meta-Stable Minima from D-branes at Obstructed Geometries

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ABSTRACT: We study the existence of long-lived meta-stable supersymmetry breaking vacua in gauge theories with massless quarks, upon the addition of extra massive flavors. A simple realization is provided by a modified version of SQCD with $N_{f,0} < N_c$ massless flavors, $N_{f,1}$ massive flavors and additional singlet chiral fields. This theory has local meta-stable minima separated from a runaway behavior at infinity by a potential barrier. We find further examples of such meta-stable minima in flavored versions of quiver gauge theories on fractional branes at singularities with obstructed complex deformations, and study the case of the dP_1 theory in detail. Finally, we provide an explicit String Theory construction of such theories. The additional flavors arise from D7-branes on non-compact 4-cycles of the singularity, for which we find a new efficient description using dimer techniques.

*Research supported by National Science Foundation Grant No. PHY-0243680

†Research supported by CICYT, Spain, under project FPA-2003-02877, and the RTN networks MRTN-CT-2004-503369 ‘The Quest for Unification: Theory confronts Experiment’ and MRTN-CT-2004-005104 ‘Constituents, Fundamental Forces and Symmetries of the Universe’.

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1. Introduction

The realization of supersymmetric gauge field theories on the world-volume of D-brane configurations in String Theory has proved to be an extremely insightful tool in the study of non-trivial gauge dynamics. In the context of $\mathcal{N} = 1$ supersymmetric gauge field theories, an interesting class of models is obtained by considering systems of D3-branes at Calabi-Yau singularities, possibly in the presence of fractional branes. The resulting quiver gauge theories lead, in the absence of fractional branes, to a tractable class of 4d strongly coupled conformal field theories, which extend the AdS/CFT correspondence [1, 2, 3] to theories with reduced (super)symmetry [4, 5, 6] and enable non-trivial precision tests of the correspondence (see for instance [7, 8])³. Quiver gauge theories constructed with both D3-branes and fractional branes are not conformal and their RG flow involves cascades of Seiberg dualities [16, 17, 18, 19, 20]. Finally, quiver gauge theories on fractional branes lead to non-conformal theories, with non-trivial strong dynamics effects, like confinement, or appearance of non-perturbative superpotentials [16, 19, 21, 22, 23].

One of the most interesting strong dynamics effects in $\mathcal{N} = 1$ supersymmetric gauge field theories, both from the theoretical and the phenomenological viewpoints, is Dynamical Supersymmetry Breaking. It is thus natural to ask whether it can be realized on the world-volume of configurations of D-branes. Strictly speaking, dynamical supersymmetry breaking requires the removal of the classical supersymmetric vacuum, and the appearance of a global non-supersymmetric minimum of the potential. D-brane configurations realizing this phenomenon have not been found yet. For instance, gauge theories on certain fractional branes at geometries without complex deformations have been shown to develop non-perturbative superpotentials which remove the supersymmetric vacuum [21, 22, 23]. However, the scalar potential of these theories, at least in the large field region where the Kähler potential can be trusted, leads to a runaway to infinity [22] (see also [24, 25]).

An interesting alternative proposal is that dynamical supersymmetry breaking occurs at local meta-stable minima, separated from supersymmetric vacua by a large potential barrier. This idea, which first appeared in phenomenological model building (see e.g. [26]) has been realized in [27] in a strikingly simple system. The authors show that the introduction of massive flavors to $SU(N)$ SYM, with masses much smaller than the dynamical scale of the gauge sector, leads to the appearance of such a local meta-stable minimum, separated from

³In the past few years there has been tremendous progress in our understanding of AdS/CFT dual pairs. In addition to the papers mentioned in the introduction, some of the works that have been crucial for these developments are [11]-[15].

the N supersymmetric minima by a potential barrier. Furthermore, the non-supersymmetric minimum can be made parametrically long-lived. We refer to this theory as the ISS model.

It is a natural question whether the introduction of extra massive flavors in more involved gauge theories also leads to such local meta-stable minima. In particular it would be interesting to explore this question for gauge theories realized on D-branes. These are the questions we address in the present paper. We find interesting generalizations of the ISS proposal, and find that the introduction of extra massive flavors leads to the appearance of local non-supersymmetric minima in diverse gauge theories with massless flavors. These theories include a simple extended version of $SU(N)$ SQCD, and the gauge theory on fractional branes on the complex cone over dP_1 (and related examples). Moreover, we argue that such local minima are likely to appear in quiver gauge theories of fractional branes in obstructed geometries (the so-called DSB fractional branes [22]).

The study of such generalization requires the development of new results in several directions, which are of interest in their own right, and which we provide in the present paper. The main results are as follows:

- Since quiver gauge theories contain massless bi-fundamentals, it is first necessary to consider the generalization of the ISS proposal to theories with massless flavors. Hence, we study the introduction of additional massive flavors to SQCD with massless flavors in detail. We show that this theory does not have local meta-stable minima at one-loop, in contrast with the ISS case.
- We consider a simple extension of SQCD with massless flavors, by introducing extra fields with cubic coupling to the flavors. We study the introduction of additional massive flavors in this theory in detail, and show the appearance of local meta-stable minima. Interestingly this extended theory, which naturally generalizes the ISS proposal to theories with massless flavors, is tantalizingly similar to the quiver gauge theories on fractional branes at obstructed geometries. It hence provides an excellent toy model of the behavior for the latter.
- We then consider the quiver gauge theory on fractional branes at the simplest example of an obstructed geometry, namely the complex cone over dP_1 (equivalent to the real cone over $Y^{2,1}$). We carry out the gauge theory analysis of this dP_1 theory upon the addition of extra massive flavors, and show the appearance of a non-trivial local minimum separated by a potential barrier from the supersymmetric minimum at infinity (equivalently, from the runaway behavior at large fields). The structure of fields and couplings, key to the existence of this minimum, is a general feature of gauge theories on fractional branes at obstructed

geometries, strongly suggesting a generalization to this full class.

- Finally, an explicit construction of this gauge theory in String Theory requires a D-brane realization of the incorporation of massive flavors. This is naturally achieved by the introduction of D7-branes in the configurations, which however has not been discussed in the literature for the case of general toric singularities. We carry out this analysis and provide new tools to introduce such D7-branes and easily determine the structure of new flavors from D3-D7 open strings, and their interactions. The flavor mass terms receive a natural interpretation in terms of vevs for higher dimensional scalars in the D7-D7 sector, which trigger a geometrical process that recombines several D7-branes, separating them from the D3-branes at the singularity.

The outcome is that gauge theories on fractional branes at obstructed geometries provide a natural generalization of the ISS proposal to quiver gauge theories. Although the computational difficulties allow us to establish this result only in particular examples, we find convincing evidence that the picture is far more general. We expect that future work in this direction confirms this expectation.

The above results are discussed in different sections. Some of them are presented as appendices to simplify the reading. The paper is organized as follows.

In Section 2 we study SQCD-like theories. Section 2.1 reviews the ISS model for SYM with extra massive flavors. Section 2.2 studies the introduction of massive flavors in SQCD with massless flavors, which does not lead to a local minimum. In Section 2.3 we describe an extension of SQCD with massless flavors, discuss its dynamics, and study the introduction of massive flavors, which in this case lead to a non-trivial, SUSY breaking minimum. We show that this minimum can be made parametrically long-lived.

Section 3 provides mostly background material. Properties of fractional branes and their quiver gauge theories are sketched in Section 3.1. Section 3.2 motivates focusing on fractional D-branes at obstructed geometries. Section 3.3 reviews the dynamics of the simplest example in this class, the dP_1 theory, in the absence of extra massive flavors.

Section 4 considers the quiver theory arising on fractional branes on the complex cone over dP_1 when massive fundamental flavors are added from a purely field theoretic perspective. This model is almost identical to the one in Section 2.3 and we show that it has a meta-stable SUSY breaking minimum. We also show that the minimum can be parametrically long-lived. Before the addition of massive flavors, this theory is the simplest example of dynamical SUSY breaking (with runaway to infinite field values) due to obstructed deformation.

Section 5 explains how to engineer the gauge theory of Section 4 using D-branes in the complex cone over dP_1 . The additional fundamental flavors are incorporated by introducing D7-branes at the singular geometry, while the flavor masses correspond to suitable vevs for D7-D7 scalars.

Appendix A describes the computation of the one-loop potential for classically flat directions in order to verify the existence or not of local SUSY breaking minima in the different theories we consider. Appendix A.1 considers SQCD with massless and additional massive flavors, Appendix A.2 studies the extended version, and Appendix A.3 describes the computation for the dP_1 theory.

Appendix B develops a general method to construct a class of D7-branes wrapping holomorphic 4-cycles in generic toric singularities and to identify their effect in the gauge theories on probe or fractional D3-branes. Appendix B.1 describes the construction for the complex cone over dP_0 , where it can be compared with orbifold techniques, since the geometry is $\mathbb{C}^3/\mathbb{Z}_3$. The rules are generalized in Appendix B.2, and applied for the complex cone over dP_1 in Appendix B.3.

2. Meta-stable vacua in $\mathcal{N} = 1$ SQCD-like theories with massive and massless flavors

In this section we first review the analysis in [27] to determine the existence of meta-stable vacua in $\mathcal{N} = 1$ $SU(N_c)$ SYM with massive flavors, and then generalize it to $SU(N_c)$ SQCD with massless and massive flavors. We show that this theory does not have a meta-stable SUSY breaking minimum. We then construct a simple modification of the model that possesses a meta-stable SUSY breaking minimum. This model constitutes an interesting proposal for SUSY breaking in theories with massless flavors. In addition, it will be an extremely useful toy model of more involved quiver gauge theories arising from D3-branes at singularities in Section 4.

2.1 $\mathcal{N} = 1$ SQCD with light massive flavors

Let us recall the system studied in [27]. Consider $SU(N_c)$ SYM with N_f massive flavors Q, \tilde{Q} with mass much smaller than Λ_{SQCD} , the dynamical scale of the gauge theory. We consider the flavor fields to have canonical Kähler potential.

The superpotential of the electric theory, for the case of equal flavor masses, is

$$W = m \operatorname{tr} \tilde{Q} Q \tag{2.1}$$

In order to have an IR free dual description, so that the Kähler potential is under control in the small field region, we require ⁴ $N_c + 1 \leq N_f < \frac{3}{2}N_c$.

The dual theory is $SU(N)$ SYM with $N = N_f - N_c$, with N_f flavors q , \tilde{q} , and mesons M . They transform as $(\square, \bar{\square}, 1)$, $(\bar{\square}, 1, \square)$ and $(1, \square, \bar{\square})$ under the $SU(N) \times SU(N_f) \times SU(N_f)$ color and flavor symmetry. The superpotential for the dual theory is

$$W = \frac{1}{\hat{\Lambda}} \text{Tr } M q \tilde{q} + m \text{Tr } M \quad (2.2)$$

where $\hat{\Lambda}$ is related to the dynamical scale Λ_{SQCD} of the electric theory and Λ of the magnetic theory by

$$\Lambda_{SQCD}^{3N_c - N_f} \Lambda^{3(N_f - N_c) - N_f} = \hat{\Lambda}^{N_f} \quad (2.3)$$

By redefining⁵ the mesons as $\Phi = M/\Lambda$ and introducing the couplings $h = \Lambda/\hat{\Lambda}$, $\mu^2 = -m\hat{\Lambda}$, the superpotential is of the form

$$W = h \text{Tr } q \Phi \tilde{q} - h\mu^2 \text{Tr } \Phi \quad (2.4)$$

(where the traces run over flavor indices). Notice that for $N_f = N_c + 1$ some further discussion is needed to establish that this superpotential correctly describes the effective dynamics, see [27] for details. A similar comment applies to all our forthcoming theories.

This theory breaks supersymmetry at tree level, since the F-flatness for Φ requires

$$\tilde{q}^i q_j = \mu^2 \delta_j^i \quad (2.5)$$

which cannot be satisfied, given that the rank of δ_j^i is N_f while the rank ⁶ of $\tilde{q}^i q_j$ is $N < N_f$. This mechanism for spontaneous SUSY breaking at tree-level has been dubbed the **rank condition mechanism** in [27]. There is a classical moduli space of minima with $V_{min} = (N_f - N)|h^2\mu^4|$, parametrized by the vevs

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix} \quad q = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad \tilde{q}^T = \begin{pmatrix} \tilde{\varphi}_0 \\ 0 \end{pmatrix}, \quad \text{with } \tilde{\varphi}_0 \varphi_0 = \mu^2 \mathbf{1}_N. \quad (2.6)$$

⁴The possibility of extending the conclusions presented in this section outside of this range has been contemplated in [27].

⁵In this simplified discussion, we ignore possible normalization factors in the Kähler potential of the fields. They can be nevertheless absorbed in additional redefinitions of flavor fields, mesons, and couplings, see [27] for details. A similar comment applies to later examples.

⁶Since the theory is IR free, the rank of $\tilde{q}^i q_j$ corresponds to its classical value.

A careful analytical computation shows that all pseudomoduli (classical flat directions not corresponding to Goldstone directions) are lifted by the one-loop effective potential, and that the maximally symmetric point in the classical moduli space

$$\Phi_0 = 0, \quad \varphi_0 = \tilde{\varphi}_0 = \mu \mathbf{1}_N, \quad (2.7)$$

is a minimum of the one-loop effective potential. The one-loop effective potential at a generic point in the classical moduli space (2.6) is the Coleman-Weinberg potential induced by the massive fluctuations around that point. We refer the reader to [27] for additional details, and to Appendix A for similar computations (in more involved situations).

The $SU(N)$ gauge dynamics is IR free and hence not relevant in the small field region, but it is crucial in the large field region. In fact, it leads to the appearance of the $N_f - N$ supersymmetric vacua predicted by the Witten index in the electric theory. In the region of large Φ vevs, $|\mu| \ll |\langle h\Phi \rangle|$, the N_f flavors get large masses due to the cubic coupling in (2.4), and we recover pure $SU(N)$ SYM dynamics, with a dynamical scale Λ' given by

$$\Lambda'^{3N} = \frac{h^{N_f} \det \Phi}{\Lambda^{N_f - 3N}} \quad (2.8)$$

where Λ is the Landau pole scale of the IR free theory. The complete superpotential, including the non-perturbative $SU(N)$ contribution is

$$W = N (h^{N_f} \Lambda^{-(N_f - 3N)} \det \Phi)^{1/N} - h\mu^2 \text{Tr } \Phi \quad (2.9)$$

This superpotential leads to $N_f - N$ supersymmetric minima at

$$\langle h\Phi \rangle = \Lambda \epsilon^{\frac{2N}{N_f - N}} \mathbf{1}_{N_f} = \mu \epsilon^{-\frac{N_f - 3N}{N_f - N}} \mathbf{1}_{N_f} \quad (2.10)$$

where $\epsilon \equiv \frac{\mu}{\Lambda}$. In the regime $\epsilon \ll 1$, the vevs are much smaller than the Landau pole scale, and the analysis can be trusted. Notice also that these minima sit at $|\langle h\Phi \rangle| \gg |\mu|$, hence at a very large distance in field space from the local non-supersymmetric minimum. This large distance, in conjunction with the height of the potential barrier separating them from the non-SUSY minimum (which can be estimated from the classical superpotential) determines that the SUSY breaking meta-stable minimum is parametrically long-lived [27].

2.2 $\mathcal{N} = 1$ SQCD with massless and massive flavors

In this section we extend the previous discussion about meta-stable vacua in $\mathcal{N} = 1$ SQCD with massive flavors [27] to another system. We investigate the case in which, in addition to light massive flavors, there are massless flavors. We consider $SU(N_c)$ SQCD with $N_{f,0}$

massless flavors \tilde{Q}_0, Q_0 and $N_{f,1}$ massive flavors \tilde{Q}_1, Q_1 , with mass much smaller than Λ_{SQCD} . Again, we consider canonical Kähler potential for these fields. To simplify the expressions, flavor indices are kept implicit. The superpotential, for the equal mass case, is thus

$$W = m \text{Tr } \tilde{Q}_1 Q_1 \quad (2.11)$$

As before, in order to have control over the computations in the IR, we consider the theory in the free magnetic range $N_c + 1 \leq N_f = N_{f,0} + N_{f,1} < \frac{3}{2}N_c$. In order for the classical theory to have SUSY breaking due to rank condition mechanism at tree level, we further require

$$N_{f,1} > N = N_{f,0} + N_{f,1} - N_c \quad \Leftrightarrow \quad N_c > N_{f,0} \quad (2.12)$$

This condition is interesting, and will reappear in Sections 3.2 and 4 in the context of branes at singularities. We now study this theory in detail since it is natural to ask whether a SUSY breaking meta-stable minimum exists. It will also serve as a warm-up for the modified model of Section 2.3.

The dual magnetic theory is $SU(N)$ SQCD with $N = N_f - N_c$, and dynamical scale Λ . There are $N_f = N_{f,0} + N_{f,1}$ flavors \tilde{q}_0, q_0 and \tilde{q}_1, q_1 , and the mesons $\Phi_{00}, \Phi_{01}, \Phi_{10}, \Phi_{11}$. The latter can be expressed as composites of the electric theory, namely $\Phi_{ij} = \frac{1}{\Lambda} \tilde{Q}_i Q_j$. The complete superpotential, in the limit where the $SU(N)$ dynamics is ignored, can be written

$$W = h \text{Tr } q \Phi \tilde{q} - h\mu^2 \text{Tr } \Phi_{11} \quad (2.13)$$

where h and μ are defined as in the theory without massless flavors in Section 2.1. Notice that for simplicity we have taken the coupling constants of the cubic terms involving Φ_{ij} , $i, j = 0, 1$, to be equal, even though no global symmetry imposes that restriction.

The equations of motion for Φ_{11} are

$$\tilde{q}_1^i q_{1,j} = \mu^2 \delta_j^i \quad (2.14)$$

Since $N_{f,1} - N = N_{f,1} - (N_{f,1} + N_{f,0} - N_c) = N_c - N_{f,0} > 0$, there is SUSY breaking by the rank condition.

Absence of a local minimum

A detailed analysis of moduli and pseudomoduli and the computation of their masses is provided in Appendix A.1. We summarize the results here.

There is a classical moduli space of degenerate supersymmetry breaking minima with $V_{min} = (N_{f,1} - N)|h^2\mu^4|$. This classical moduli space can be parametrized as follows

$$\begin{aligned} q_0 = \tilde{q}_0 = 0 \quad q_1 = (\varphi_1; 0) \quad \tilde{q}_1 = \begin{pmatrix} \tilde{\varphi}_1 \\ 0 \end{pmatrix} \\ \Phi_{01} = (0; Y) \quad \Phi_{10} = \begin{pmatrix} 0 \\ \tilde{Y} \end{pmatrix} \quad \Phi_{11} = \begin{pmatrix} 0 & 0 \\ 0 & X_1 \end{pmatrix} \end{aligned} \quad (2.15)$$

where $\tilde{\varphi}_1, \varphi_1$ are $N \times N$ blocks satisfying $\tilde{\varphi}_1\varphi_1 = \mu^2 \mathbf{1}_N$. In addition, Y, \tilde{Y} and X_1 are $N_{f,0} \times (N_{f,1} - N)$, $(N_{f,1} - N) \times N_{f,0}$ and $(N_{f,1} - N) \times (N_{f,1} - N)$ blocks, respectively. The vev for $\Phi_{00} = X_0$ is arbitrary.

Goldstone bosons corresponding to broken global symmetries remain exactly massless. Integrating out classically massive fluctuations, the one-loop effective potential becomes

$$\langle V_{eff}^{(1)} \rangle = const. + |h^4\mu^2| \frac{(\log 4 - 1)}{16\pi^2} N \left[(N_f - N) \left(2 |\delta\Phi_1|^2 + |\mu^2|(\theta + \theta^*)^2 \right) + \tilde{N} \left(|\delta Y|^2 + |\delta\tilde{Y}|^2 \right) \right] + \dots \quad (2.16)$$

The variables θ , etc are defined in (A.6) and $\tilde{N} = \min(N_{f,0}, N_{f,1} - N)$. This expression assumes that the non-vanishing parts of Y and \tilde{Y} are proportional to the identity. See (A.9) for a slightly more general equation.

We see that $\delta\Phi_0$ remains massless at 1-loop. In principle, it is still possible that $\delta\Phi_0$ becomes massive at higher loops, producing a meta-stable minimum (probably with a much smaller potential barrier) at small expectation values for the fields. We will not consider this possibility, but will explore an extension of this model for which this flat direction is lifted at the classical level in next section.

Behavior at large fields

In analogy with the case in Section 2.1, we expect that in the large field region we recover the low-energy structure of the $SU(N_c)$ theory with $N_{f,0} < N_c$ flavors, namely an Affleck-Dine-Seiberg (ADS) superpotential triggering a runaway behavior for the meson $\Phi_0 = \tilde{Q}_0 Q_0$ of the electric theory. This is indeed the case, and we now show it from the perspective of the magnetic theory.

Consider a generic point in the moduli space of the magnetic theory with non-vanishing expectation values of Φ_{00} and Φ_{11} . The flavors \tilde{q}_0 and q_0 become massive, with mass matrix $h\Phi_{00}$. Similarly, \tilde{q}_1 and q_1 get masses given by $h\Phi_{11}$. We can then integrate out these fields, solving their equations of motion by setting $\Phi_{10} = \Phi_{01} = 0$. The resulting theory is pure $SU(N)$ with a dynamical scale given by

$$\Lambda'^{3N} = \frac{h^{N_f} \det \Phi_{00} \det \Phi_{11}}{\Lambda^{N_f-3N}} \quad (2.17)$$

The complete superpotential, including the non-perturbative piece, reads

$$W = -h\mu^2 \text{Tr} \Phi_{11} + N \left(\frac{h^{N_f} \det \Phi_{00} \det \Phi_{11}}{\Lambda^{N_f-3N}} \right)^{\frac{1}{N}} \quad (2.18)$$

Recall that Φ_{00} is (up to a rescaling by $1/\Lambda$) equal to the meson Φ_0 of the original electric theory, so we are interested in its dynamics. The effective action for Φ_{00} can be obtained by using the equation of motion for Φ_{11} . Integrating out Φ_{11} we obtain

$$W = -(N_{f,1} - N) \left(\frac{\mu^{2N_{f,1}} \Lambda^{N_f-3N}}{h^{N_{f,0}} \det \Phi_{00}} \right)^{\frac{1}{N_{f,1}-N}} \quad (2.19)$$

Noticing that $N_{f,1} - N = N_c - N_{f,0}$ in terms of parameters of the electric theory, this is exactly the runaway superpotential for Φ_0 induced by $SU(N_c)$ SQCD with $N_{f,0}$ massless flavors. The unfamiliar structure of factors inside the bracket is simply due to the fact that the $SU(N_c)$ dynamical scale appears expressed in terms of the Landau pole scale of the magnetic theory.

Notice that the runaway potential can be trusted as long as $|h\langle\Phi_{00}\rangle| \ll \Lambda$. For larger fields, the electric theory completes the UV and ensures that the runaway persists to infinity.

As described in Section 2.2 and Appendix A.1, the one-loop potential for this theory leaves the field Φ_{00} massless around the origin, and moreover slopes down as this field increases. Hence, Φ_{00} is not stabilized in the small vev region. At large fields, we have a runaway to infinity for this field. The most conservative proposal is thus to connect these two behaviors in a constantly decreasing potential in the direction Φ_{00} . Of course higher loop contributions could in principle lead to a non-trivial behavior in the intermediate field regime. Since this is however difficult to establish, in the next section we turn to the study of a different theory, which incorporates a mild extension of the above model. We will show that this new theory does have a meta-stable minimum separated from a runaway behavior at infinity by a potential barrier. In addition, the extension brings the model closer to quiver gauge theories with obstructed deformations, which are studied in later sections.

2.3 Extension of SQCD with massless flavors

As discussed in the previous section, SQCD with massless and massive flavors does not have a meta-stable SUSY breaking minimum at one-loop. In this section we propose a simple

extension of this theory that does have such meta-stable SUSY breaking minimum, which is separated from a runaway behavior at infinity by a potential barrier and is parametrically long-lived.

The extension amounts to the introduction of a new field Σ_0 , with cubic coupling to the flavors of the original electric theory. The role of this extension in leading to a meta-stable vacuum is easily understood. In the dual theory, the field Σ_0 couples to the meson Φ_{00} via a mass term, forcing the vev of the latter to vanish. Hence the new term eliminates the Φ_{00} direction which was not properly lifted by the one-loop potential.

The extended theory without massive flavors

Let us start by describing the extended theory and its dynamics in the absence of massive flavors. Consider $SU(N_c)$ SQCD with $N_{f,0} < N_c$ massless flavors \tilde{Q}_0, Q_0 and add a set of singlets Σ_0 , transforming in the bi-fundamental of the $SU(N_{f,0})^2$ flavor global symmetry, with a superpotential

$$W_{ext} = g \text{Tr } Q_0 \Sigma_0 \tilde{Q}_0 \quad (2.20)$$

where g is a dimensionless coupling⁷. We consider canonical Kähler potentials for all fields.

This theory has a runaway behavior in the new field Σ_0 . To show this, we introduce the gauge invariant mesons⁸ $\Phi_0 = Q_0 \tilde{Q}_0$. The complete superpotential is

$$W = g \text{Tr } \Sigma_0 \Phi_0 + (N_c - N_{f,0}) \left(\frac{\Lambda_{SQCD}^{3N_c - N_{f,0}}}{\det \Phi_0} \right)^{\frac{1}{N_{f,0} - N_c}} \quad (2.21)$$

Upon using the equation of motion for Φ_0 we have

$$W = N_c \left(g^{N_{f,0}} \Lambda_{SQCD}^{3N_c - N_{f,0}} \det \Sigma_0 \right)^{\frac{1}{N_c}} \quad (2.22)$$

This is indeed a runaway behavior: the F-term for Σ_0 gives

$$\frac{\partial W}{\partial (\Sigma_0)_{ij}} \simeq (\det \Sigma_0)^{1/N_c} (\Sigma_0^{-1})_{ji} \quad (2.23)$$

which, scaling $\Sigma_0 \rightarrow \lambda \Sigma_0$, scales as $\lambda^{\frac{N_{f,0} - N_c}{N_c}}$. Hence since $N_{f,0} < N_c$, the F-terms relax to zero for large fields. With some foresight, we note that this behavior is completely analogous to the one we will discuss in Section 3.3 for the dP_1 quiver gauge theory.

⁷This model has been discussed in [28], where it was called SSQCD (for singlets + SQCD). In that paper, the IR phases of this theory were studied using Seiberg duality and a-maximization.

⁸Although almost identical, this meson differs slightly from the Φ_{00} we defined in Section 2.2. The latter was an elementary field in the magnetic theory, so we included a power of Λ in its definition to give it canonical dimensions.

Introducing massive flavors in the extended theory

The above theory is a close cousin of SQCD, and in particular it shares its runaway behavior (albeit in a different field direction). However, as we discuss later, they differ in their dynamics when extra light massive flavors are introduced. In particular, the extended theory will show meta-stable minima.

The extended theory with massive flavors is a combination of the SQCD with massless and massive flavors of Section 2.2 and the extension term introduced above. Hence we consider $SU(N_c)$ SQCD with $N_{f,0}$, $N_{f,1}$ massless and massive flavors, and fields Σ_0 , coupled to the massless flavors via (2.20). As in Section 2.2, we consider $N_{f,0} < N_c$ and hence $N_{f,1} > N$, so that the dual theory has supersymmetry breaking by the rank condition at tree level.

The dual magnetic theory is $SU(N)$ SQCD with $N = N_f - N_c$, and $N_f = N_{f,0} + N_{f,1}$ flavors, with dynamical scale Λ . We also have mesons $\Phi_{ij} = \frac{1}{\Lambda} \tilde{Q}_i Q_j$, and the classical superpotential

$$W = h \text{Tr } q \Phi \tilde{q} - h\mu^2 \text{Tr } \Phi_{11} + h\mu_0 \text{Tr } \Sigma_0 \Phi_{00} \quad (2.24)$$

where $h = \Lambda/\hat{\Lambda}$, $\mu^2 = -m\hat{\Lambda}$, $\mu_0 = g\Lambda$, and $\hat{\Lambda}$, Λ are related to the electric scale Λ_{SQCD} by (2.3).

As usual the equations of motion for Φ_{11} lead to SUSY breaking by the rank condition.

The local minimum

Although (2.20) is a simple modification of (2.13), the addition of the new field and its interactions has a drastic effect in the small field region of the theory. A full discussion of pseudomoduli and their masses in this theory is given in Appendix A.2. The classical SUSY breaking minima are parametrized as in (2.15), with X_0 fixed to zero by the tree-level superpotential (i.e. X_0 is no longer a pseudomodulus).

The one-loop effective potential has a critical point at $\Phi_1 = Y = \tilde{Y} = (\theta + \theta^*) = 0$. Around this point, it becomes

$$\langle V_{eff}^{(1)} \rangle = \text{const.} + |h^4 \mu^2| \frac{(\log 4 - 1)}{16\pi^2} N \left[(N_f - N) \left(2 |\delta\Phi_1|^2 + |\mu^2|(\theta + \theta^*)^2 \right) + \tilde{N} \left(|\delta Y|^2 + |\delta \tilde{Y}|^2 \right) \right] + \dots \quad (2.25)$$

Hence, all pseudomoduli get positive masses and are thus stabilized. The critical point becomes a meta-stable minimum, whose longevity we analyze later. Again the expression above corresponds to the non-vanishing parts of Y and \tilde{Y} being proportional to the identity. Equation (A.12) gives the general result.

Behavior at large fields

In the large field region of the theory (2.24), we expect to recover the behavior in the absence of massive flavors. Namely, we expect a runaway of Σ_0 dictated by (2.22). This is indeed the case as we now show.

We can study the large field region by considering the expression (2.19), which describes the large field behavior for the non-extended theory, and adding the extension term $-h\mu_0\text{Tr}\Phi_0\Sigma_0$. Upon integrating out Φ_{00} , we obtain

$$W = -(N_f - N) \left(\mu_0^{N_f,0} \mu^{2N_f,1} \Lambda^{N_f-3N} \det \Sigma_0 \right)^{\frac{1}{N_f-N}} \quad (2.26)$$

Recalling that $N_f - N = N_c$ in terms of the underlying electric theory, this behavior⁹ is essentially identical to the runaway of the extended theory without the massive flavors (2.22). The different factors inside the bracket are simply due to expressing the superpotential in terms of the Landau pole scale of the magnetic theory.

Lifetime of meta-stable vacua

The decay rate is proportional to the semi-classical decay probability. This probability is proportional to $\exp(-S)$, where the bounce action S is the difference in the Euclidean action between the tunneling configuration and the meta-stable vacuum.

The SUSY breaking, meta-stable vacuum is given by

$$q_1 = (\varphi_1; 0) \quad \tilde{q}_1 = \begin{pmatrix} \tilde{\varphi}_1 \\ 0 \end{pmatrix} \quad (2.27)$$

with $\tilde{\varphi}_1 = \varphi_1 = \mu \mathbf{1}_N$, and all the other fields having a zero expectation value.

We saw in the previous section that the SUSY vacuum corresponds to Σ_0 running away to infinity due to the superpotential (2.26). Simultaneously, the equations of motion force the fields Φ_{00} and Φ_{11} to have non-zero vevs, adjusted to the Σ_0 vev.

In order to estimate the bounce action, we must find a trajectory in field space connecting the meta-stable and SUSY vacua such that the potential barrier is minimum. The classical superpotential (2.24) does not have any coupling that would give rise to contributions to the classical scalar potential of the form $|h^2\varphi_1\Sigma_0|^2$ or $|h^2\tilde{\varphi}_1\Sigma_0|^2$. Such contributions would become very large as Σ_0 runs away if φ_1 or $\tilde{\varphi}_1$ do not vanish. Anyway, it is convenient

⁹Notice that, despite the fact that $\Phi_{00}, \Phi_{11} \rightarrow 0$ as Σ_0 runs to infinity, for each fixed value of Σ_0 the flavor masses remain parametrically larger (for $\epsilon = \mu/\Lambda \ll 1$, $\alpha_\mu = \mu_0/\mu \gg 1$) than the vacuum energy, hence it is consistent to keep them integrated out, as implicitly done in our computation.

to consider the following simple trajectory connecting both vacua, which exhibits the characteristic barrier height and distance in field space separating the meta-stable and SUSY vacua: first go to the origin, where the potential is

$$V_T = N_{f,1} |h^2 \mu^4| \quad (2.28)$$

and then approach the SUSY vacuum at infinity increasing Σ_0 .

To estimate S , we model the potential as a triangular barrier, for which the exact bounce action has been derived in [29]. A triangular barrier has the general form depicted in Figure 1.

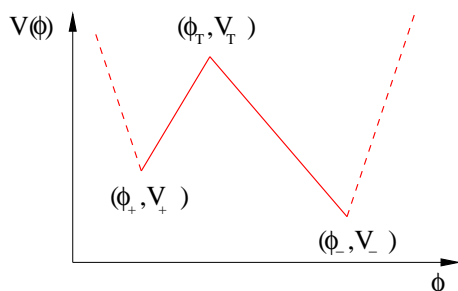


Figure 1: A triangular potential barrier.

We define the quantities $\Delta\phi_{\pm} = \pm(\phi_T - \phi_{\pm})$ and $\Delta V_{\pm} = (V_T - V_{\pm})$. A triangular barrier is a good approximation in cases in which the gradient of the potential is approximately constant at both sides of the peak. In this case,

$$S \sim \frac{|(\Delta\phi_+)^2 - (\Delta\phi_-)^2|^2}{\Delta V_+} \quad (2.29)$$

Modeling the barrier should be done in slightly different way from the SQCD with light flavors case [27], since in this case there is a runaway and the potential does not vanish at finite values of the fields. The slope of our potential becomes progressively smaller as $\Sigma_0 \rightarrow \infty$. A cartoon of the potential is presented in Figure 2, showing the criterion we will use to define the triangular barrier.

We should interpret the variable ϕ in Figure 2 as parametrizing the trajectory in field space connecting the meta-stable and SUSY vacua. Hence, the region before the peak corresponds to motion in φ_1 and $\tilde{\varphi}_1$, and the region after it corresponds to motion in the Σ_0 direction. In addition, Figure 2 is not drawn at scale, and the distance between the meta-stable minimum and the peak is negligible with respect to the rest of the plot.

Since the potential does not vanish, but asymptotes zero, we define ϕ_- as the point in the large field region at which the potential falls below $|h^2 \mu^4|$. The height of the barrier

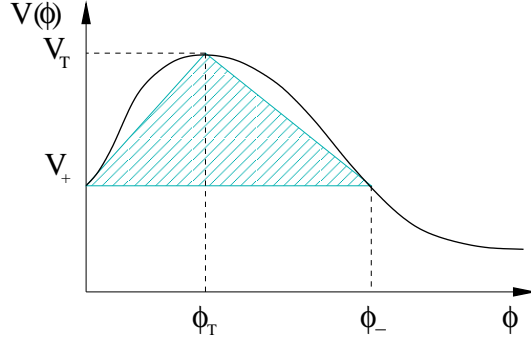


Figure 2: Sketch of the potential along the bounce trajectory and the triangular barrier we use to model it.

is also of order $|h^2\mu^4|$. At the level of the estimations we are making, the calculation we perform would also correspond to an alternative criterion: assuming that $V(\phi_-) = 0$ (for ϕ_- as just defined) thus underestimating the barrier and producing a lower bound for the bounce action.

Taking the simple ansatz $\Sigma_0 = \sigma_0 \mathbf{1}_{\mathbf{N}_{f,0}}$, we obtain

$$V \sim \left| \mu^{2N_{f,1}} \mu_0^{N_{f,0}} \Lambda^{N_f-3N} \sigma_0^{N-N_{f,1}} \right|^{\frac{2}{N_f-N}} \quad (2.30)$$

Then, in order to have $V \sim |h^2\mu^4|$, we have

$$|\sigma_{0-}| \sim \left| \frac{\Lambda^{N_f-3N} \mu_0^{N_{f,0}}}{h^{N_f-N} \mu^{2(N_{f,0}-N)}} \right|^{\frac{1}{N_{f,1}-N}} \quad (2.31)$$

In our case, $\Delta\phi_- \sim \mu$ is negligible, $\Delta V_+ \sim |h^2\mu^4|$ and $\Delta\phi_- \sim \sigma_{0-}$. Using (2.29) we obtain

$$S \sim \frac{1}{|h|^{6+4N_{f,0}/(N_f-N)}} \frac{|\alpha_\mu|^{4N_{f,0}/(N_{f,1}-N)}}{|\epsilon|^{4(N_f-3N)/(N_{f,1}-N)}} \quad (2.32)$$

where we have defined $\epsilon = \mu/\Lambda$ as in Section 2.1 and $\alpha_\mu = \mu_0/\mu$ measures the strength of the extension term relative to the rest of the superpotential. The general behavior is clear: for a fixed value of the coupling h the lifetime can be made parametrically large by either making the fundamental flavors in the original theory light (i.e. small ϵ) or by increasing the relative strength of the extension term in the superpotential (given by α_μ). A heuristic reason for the latter is that α_μ indicates how much the extension term “pushes” the system into a runaway in the Σ_0 direction.

For $N_{f,0} = 0$ (2.32) becomes

$$S \sim \frac{1}{|h|^6} \frac{1}{|\epsilon|^{4(N_{f,1}-3N)/(N_{f,1}-N)}} \quad (2.33)$$

The result of [27] is identical to this one except that the power of $|\epsilon|$ is $4(N_{f,1} - 2N)/(N_{f,1} - N)$, i.e. larger, in that case. This discrepancy is precisely accounted for by noticing that our criterion for determining the potential barrier underestimates $\Delta\phi_-$, and hence the bounce action, with respect to [27].

3. Review of fractional branes and obstructed deformations

In this section we describe quiver gauge theories based on D3-branes at singularities with fractional branes. We point out that the gauge theories on the so-called ‘DSB fractional branes’ have features analogous to the extended version of SQCD with massless flavors studied above. This will motivate the study of these theories with additional massive flavors in coming sections.

3.1 General review of fractional branes

D-branes at singularities provide a useful arena to study and test the gauge/string correspondence, in situations with reduced (super)symmetry. In particular, the introduction of fractional branes leads to interesting dual pairs involving non-conformal gauge theories with non-trivial dynamics in the infrared. In the string construction, fractional branes correspond to D-branes wrapped on cycles collapsed at the singularity, consistently with cancellation of (local) RR tadpoles. At the level of the gauge theory, fractional branes correspond to rank assignments for gauge factors in a way consistent with cancellation of non-abelian anomalies.

A particularly well-known class of systems corresponds to D3-branes at toric singularities. The corresponding gauge theories are described in terms of brane tiling or dimer graphs [30, 31, 32, 33, 34]. We restrict to this class in what follows, although some facts are valid in non-toric singularities as well.

A classification of different kinds of fractional branes, the infrared behavior of the associated gauge theory, and corresponding features in the geometry, is as follows [22]:

- **$\mathcal{N} = 2$ fractional branes:** these are fractional branes whose quiver gauge theory (in the absence of D3-branes) corresponds to a closed loop of arrows passing through a set of nodes, with the associated gauge invariant operator *not* appearing in the superpotential. These fractional branes therefore have flat directions, parametrized by vevs for this mesonic operator, and along which the effective theory is $\mathcal{N} = 2$ supersymmetric. Geometrically,

these fractional branes exist for singularities which are not isolated, but have (complex) curves of $\mathbb{C}^2/\mathbb{Z}_N$ singularities passing through them. The fractional branes correspond to D5-branes wrapped on the 2-spheres collapsed at the latter. The prototypical example is provided by branes at the $\mathbb{C}^2/\mathbb{Z}_2$ singularity. In the gauge/gravity description, the IR dynamics of the gauge theory (instantons and Seiberg-Witten points) corresponds to an enhançon behavior on the gravity side.

- **Deformation fractional branes:** these are fractional branes whose quiver is either given by a set of decoupled nodes, or by a set of nodes joined by a closed loop of arrows, with the corresponding gauge invariant operator appearing in the superpotential. Moreover, the involved gauge factors all have the same rank. Geometrically, these fractional branes are associated with a possible complex deformation of the singularity. These are easily described in terms of splitting of the web diagram [35, 36, 37] of the singularity into sub-webs. The prototypical example is provided by branes at the conifold singularity. The behavior of the gauge theory corresponds to confinement of the involved gauge groups, and in the dual gravity background this corresponds to a complex deformation leading to finite size 3-cycles.

- **DSB fractional branes:** these are fractional branes of any other kind, hence they provide the generic case. They are fractional branes for which the non-trivial gauge factors have unequal ranks. Geometrically, they are associated with obstructed geometries, which do not admit the corresponding complex deformation¹⁰. As discussed in [21, 22, 23], the dynamics of the gauge theory corresponds to the appearance of an ADS superpotential which removes the supersymmetric minimum. Moreover, as first discussed in [22] and later studied in detail in [24, 25]) the theory has a runaway behavior towards infinity (in a direction parametrized by di-baryonic operators), at least in the large field regime. The absence of a vacuum at finite values of the fields suggests that the dual supergravity background, describing the UV behavior of the theory may not admit a smoothing of their naked singularities. The prototypical case is the fractional brane of the complex cone over dP_1 , which we study in next section.

3.2 Quiver gauge theories and the ISS proposal

We would like to consider the possible generalization of the ISS proposal to quiver gauge theories on fractional branes. Notice that adding massive flavors to $N = 2$ SYM was shown

¹⁰An important and often unnoticed fact, which has been discussed in [22], is that geometries admitting complex deformations may have DSB fractional branes, since generically the number of complex deformations is smaller than the number of independent fractional branes. An example is provided by the complex cone over dP_3 , which admits two complex deformations and three independent fractional branes.

in [27] not to lead to SUSY breaking local meta-stable minima. Hence $N = 2$ fractional branes are not appropriate candidates to implement the ISS proposal.

On the other hand, one can consider deformation branes leading to a set of decoupled $\mathcal{N} = 1$ SYM theories in the infrared, the simplest case being the fractional brane of the conifold theory. Addition of flavors to these theories leads to a direct realization of the ISS model, so the analysis in [27] goes through without modification. Thus, these are the simplest examples of D-brane configurations realizing the ISS proposal in the D-brane. Notice however that deformation branes whose quiver gauge theory reduces simply to $\mathcal{N} = 1$ SYM exist only for very non-generic cases, like vector-like theories.

Hence to understand the generic extension of the ISS proposal to quiver gauge theories we have to consider the remaining cases. They correspond to more general deformation fractional branes (leading to a set of nodes joined by arrows, like for instance the three-node fractional brane of the dP_3 theory studied in [19]), or DSB fractional branes. All these gauge theories contain massless bi-fundamentals, hence the relevant version of the ISS proposal is that provided in Section 2.3. For the ISS proposal to have a chance to work, some necessary conditions are required. First, the theory after dualizing the node in the free magnetic phase, should have SUSY breaking by the rank condition. As in Sections 2.2, 2.3, this requires $N_{f,1} > N_c$, equivalently $N_{f,0} < N_c$. This condition is not satisfied by deformation fractional branes of the kind we are considering (namely, with quivers given by equal rank nodes joined by arrows). But it is satisfied for DSB branes, on which we center henceforth. A second condition is the existence of suitable fields with cubic couplings to the massless flavors of the node in the free magnetic phase. It is easy to verify that in all known examples of DSB branes this condition is satisfied as well.

Hence DSB fractional branes are the natural setup to provide a generalization of the ISS proposal for generic quiver gauge theories. In the remainder of the paper, we focus on systems of DSB fractional branes, in the absence of D3-branes. These theories can be regarded and studied on their own. Alternatively, one can consider them as the IR result of a duality cascade, along which the D3-branes have disappeared. Some remarks about the latter interpretation, and the related issue of gravity duals of the gauge theory phenomena we discuss, are presented in Section 6.

3.3 Review of the runaway for fractional branes in the cone over dP_1

We now focus on DSB fractional branes. In this section we review the dynamics in the absence of additional massive flavors, while in the next one we consider the possible appearance of meta-stable vacua once extra flavors are included. Both in this and coming sections we center

on the simplest situation of the fractional brane of the complex cone over dP_1 . However, we expect that much of our discussion is valid for the general case, and in fact our analysis is automatically valid for other examples of fractional branes on obstructed geometries leading to the same quiver gauge theory (for instance, the fractional branes of dP_5 studied in [39]).

The gauge theory on D3-branes at a singularity given by a complex cone over dP_1 was determined in [38]. The gauge theory on a set of fractional branes has quiver shown in Figure 3. For convenience we have labeled the nodes such that the gauge factor associated with a node with label k is $SU(kM)$. Please note that this convention is different from others in the literature.

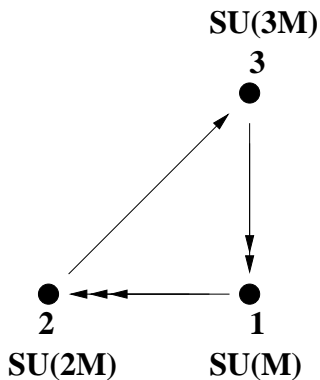


Figure 3: Quiver diagram for M fractional branes in the complex cone over dP_1 .

The superpotential is given by

$$W = \lambda (X_{23}X_{31}Y_{12} - X_{23}Y_{31}X_{12}) \quad (3.1)$$

with obvious notation. Here and in what follows, traces over color indices are implicit. We have introduced a dimensionless coupling λ , which is equal for both terms since they are related by an $SU(2)$ global symmetry.

This theory develops a non-perturbative superpotential which removes the supersymmetric minimum [21, 22, 23]. Indeed, the theory has a runaway towards infinity ¹¹ [22], see [24] for a detailed discussion. The runaway can be easily seen in both the $SU(2M)$ and $SU(3M)$ dominated regimes. The computation is very similar in both cases, and we now review the situation in which the $SU(3M)$ dynamics dominates. Since this gauge factor confines, we construct the mesons

$$M_{21} = X_{23}X_{31} \quad ; \quad M'_{21} = X_{23}Y_{31} \quad (3.2)$$

¹¹Notice that this statement requires some assumption about the Kähler potential for the dP_1 theory.

The $SU(3M)$ gauge factor has $2M$ flavors, and leads to a non-perturbative ADS superpotential. The full superpotential is

$$W = \lambda (M_{21} Y_{12} - M'_{21} X_{12}) + M \left(\frac{\Lambda_3^{7M}}{\det \mathcal{M}} \right)^{\frac{1}{M}} \quad (3.3)$$

where $\mathcal{M} = (M_{21}; M'_{21})$ is the mesonic $2M \times 2M$ matrix.

For simplicity let us focus on the case $M = 1$, and denote

$$M_{21} = \begin{pmatrix} A \\ C \end{pmatrix} \quad ; \quad M'_{21} = \begin{pmatrix} B \\ D \end{pmatrix} \quad ; \quad Y_{12} = (a, b) \quad ; \quad X_{12} = (c, d) \quad (3.4)$$

We then have

$$W = \lambda (aA + bC - cB - dD) + \frac{\Lambda_3^7}{AD - BC} \quad (3.5)$$

Using the equations of motion for A, B, C, D we obtain

$$W \simeq (\lambda^2 \Lambda_3^7 \det \mathcal{Y})^{1/3} \quad (3.6)$$

where $\mathcal{Y} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} Y_{12} \\ X_{12} \end{pmatrix}$ and we have dropped an unimportant numerical factor. This superpotential leads to a runaway behavior¹² for the fields X_{12}, Y_{12} . Notice that these fields are not the mesons of the confining group, but rather the microscopic fields which had cubic couplings with the original flavors.

Along this direction in field space, the additional gauge symmetry $SU(2M) \times SU(M)$ is generically Higgsed by the vevs for \mathcal{Y} , hence it does not lead to any modifications of the above behavior.

The conclusion is that the theory has a runaway in the direction \mathcal{Y} corresponding to the singlets (of the strong dynamics gauge factor) with cubic couplings with the flavors X_{32}, X_{13}, Y_{13} . This behavior is reminiscent of that of the extended version of SQCD studied in Section 2.3. This analogy suggests that the dP_1 theory may lead to meta-stable minima upon the addition of extra massive flavors. In the next section we add massive fundamental flavors to the theory, and indeed find the appearance of a meta-stable SUSY breaking minimum.

4. Flavored dP_1

Inspired by the ideas presented in previous sections, we consider the dP_1 theory and explore whether the addition of light massive flavors for node 3 can lead to a long-lived, meta-stable,

¹²Again, notice that the existence of a runaway scalar potential implies certain assumptions for the Kähler potential.

SUSY-breaking minimum ¹³. The String Theory construction leading to the additional fundamental flavors is provided in Section 5.

4.1 The classical flavored dP_1 theory

In the String Theory construction, see next section, we discuss that light massive fundamental flavors can be introduced by adding D7-branes in the configuration, with the new flavors arising from open strings stretched between D3 and D7-branes. Consistent sets of D7-branes typically add the same number of flavors to all gauge factors in the quiver.

As discussed in Section 5, there are several different choices of a consistent set of D7-branes that can be added. These different choices lead in general to the same flavor content for the different gauge factors (i.e. the same quiver), but differ in the interactions of the latter with the D3-D3 states ¹⁴. Specifically, different D7-branes lead to D3-D7 (and D7-D3) states with cubic coupling to different D3-D3 bifundamentals.

In order to keep the discussion concrete, we center on a specific set of D7-branes. Other choices can be analyzed similarly. We consider three kinds of D7-branes, whose D3-D7, D7-D3 states couple to the 33 fields X_{23} , X_{31} and X_{12} respectively. In general we will consider $N_{f,1}$ copies of this set of D7-branes, labeled by indices i, j, k , leading to $N_{f,1}$ additional flavors for each D3-brane gauge factor. The resulting gauge theory can be encoded in an **extended quiver**, with additional nodes representing gauge symmetries on the D7-branes, and additional arrows representing the new flavors. The gauge fields on the D7-brane world-volume are higher-dimensional and thus appear on the four-dimensional theory as flavor global symmetries. The extended quiver diagram for this gauge theory with flavors is shown in Figure 4, where D7-branes are represented as white nodes. Our notation is that Q_{ai} , \tilde{Q}_{ia} denote flavor fields associated with the a^{th} D3-brane gauge factor and a D7-brane in the i^{th} set. Notice that each of the indices for the three kinds of D7-branes can be regarded as an independent $SU(N_{f,1})$ global symmetry group. However, mass terms to be introduced later will break this symmetry, in general to a diagonal combination.

In addition to the superpotential (3.1), we have a superpotential for the new flavors

$$W_{flav.} = \lambda' (Q_{3i}\tilde{Q}_{i2}X_{23} + Q_{2j}\tilde{Q}_{j1}X_{12} + Q_{1k}\tilde{Q}_{k3}X_{31}) \quad (4.1)$$

¹³As mentioned before, our analysis automatically generalizes to other examples of fractional branes on obstructed geometries leading to the same quiver gauge theory (for instance, the fractional branes of dP_5 studied in [39]).

¹⁴Notice that, following the abuse of language of appendix B, in this discussion what we mean by ‘D3-brane’ is a gauge factor in the quiver theory. Such a gauge factor can arise from either regular or fractional D3-branes (wrapped D5-branes).

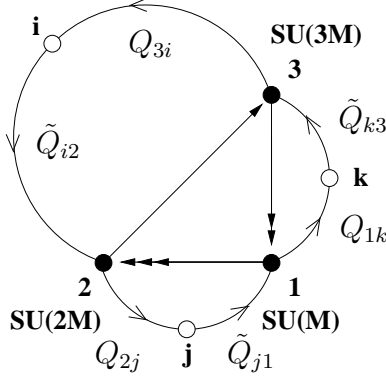


Figure 4: Extended quiver diagram for a dP_1 theory with flavors.

where for simplicity we assume the same coupling λ' for all terms. We also introduce mass terms

$$W_m = m_3 Q_{3i} \tilde{Q}_{k3} \delta_{ik} + m_2 Q_{2j} \tilde{Q}_{i2} \delta_{ji} + m_1 Q_{1k} \tilde{Q}_{j1} \delta_{kj} \quad (4.2)$$

Although we work with independent masses, the general results are valid in the simpler situation of equal masses. Note that since the mass terms mix the global symmetries of the different D7-branes, we stop using different indexes for them.

We would like to introduce a number $N_{f,1}$ of flavors such that the $SU(3M)$ node is in the free magnetic phase. This corresponds to $N_c + 1 \leq N_f < \frac{3}{2}N_c$. Since $N_f = N_{f,0} + N_{f,1}$ with $N_{f,0} = 2M$, we require

$$M + 1 \leq N_{f,1} < \frac{5}{2}M \quad (4.3)$$

A simple choice which works for all M , including $M = 1$, is $N_{f,1} = 2M$. For the moment we keep M and $N_{f,1}$ general.

Let us perform a Seiberg duality transformation on node 3. The dual gauge factor is $SU(N)$ with $N = N_{f,1} - M$, and dynamical scale Λ . To get the matter content in the dual, we replace the microscopic flavors Q_{3i} , \tilde{Q}_{k3} , X_{23} , X_{31} , Y_{31} by the dual flavors \tilde{Q}_{i3} , Q_{3k} , X_{32} , X_{13} , Y_{13} . We also have the mesons, related to the fields in the electric theory by

$$\begin{aligned} M_{21} &= \frac{1}{\Lambda} X_{23} X_{31} & N_{k1} &= \frac{1}{\Lambda} \tilde{Q}_{k3} X_{31} \\ M'_{21} &= \frac{1}{\Lambda} X_{23} Y_{31} & N'_{k1} &= \frac{1}{\Lambda} \tilde{Q}_{k3} Y_{31} \\ N_{2i} &= \frac{1}{\Lambda} X_{23} Q_{3i} & \Phi_{ki} &= \frac{1}{\Lambda} \tilde{Q}_{k3} Q_{3i} \end{aligned} \quad (4.4)$$

There is a cubic superpotential coupling the mesons and the dual flavors

$$\begin{aligned} W_{mes.} &= h (M_{21} X_{13} X_{32} + M'_{21} Y_{13} X_{32} + N_{2i} \tilde{Q}_{i3} X_{32} + \\ &+ N_{k1} X_{13} Q_{3k} + N'_{k1} Y_{13} Q_{3k} + \Phi_{ki} \tilde{Q}_{i3} Q_{3k}) \end{aligned} \quad (4.5)$$

where we have taken a common coupling $h = \Lambda/\hat{\Lambda}$, with $\hat{\Lambda}$ related to Λ , Λ_3 by the analog of (2.3).

In addition we have the classical superpotential, written in terms of the new fields

$$W_{clas.} = h\mu_0 (M_{21}Y_{12} - M'_{21}X_{12}) + \mu' Q_{1k}N_{k1} + \mu' N_{2i}\tilde{Q}_{i2} + \\ - h\mu^2 \text{Tr } \Phi + \lambda' Q_{2j}\tilde{Q}_{j1}X_{12} + m_2 Q_{2i}\tilde{Q}_{i2} + m_1 Q_{1i}\tilde{Q}_{i1} \quad (4.6)$$

where $\mu_0 = \lambda\Lambda$, $\mu' = \lambda'\Lambda$, and $\mu^2 = -m_3\hat{\Lambda}$. Although not manifest in our present notation, some of the fields in this theory are close analogs of fields in the extended SQCD model in Section 2.3. We clarify this analogy in Appendix A.3.

Some of the fields are massive, so we will proceed to integrating them out. However recall that M_{21} , M'_{21} , X_{12} , Y_{12} play a crucial role in the dynamics of the un-flavored dP_1 theory. In fact, they are the analogs of Φ_{00} and Σ_0 in the extension of SQCD with massless and massive flavors. Hence it is convenient to keep them in the effective action until the last stage of the analysis. Thus we integrate out \tilde{Q}_{i2} , N_{2i} , Q_{1k} , N_{k1} .

The resulting superpotential is

$$W = h\Phi_{ki}\tilde{Q}_{i3}Q_{3k} - h\mu^2 \text{tr } \Phi + h\mu_0 (M_{21}Y_{12} - M'_{21}X_{12}) + \\ + h (M_{21}X_{13}X_{32} + M'_{21}Y_{13}X_{32} + N'_{k1}Y_{13}Q_{3k}) + \\ + \lambda' Q_{2j}\tilde{Q}_{j1}X_{12} - h_1 \tilde{Q}_{k1}X_{13}Q_{3k} - h_2 Q_{2i}\tilde{Q}_{i3}X_{32} \quad (4.7)$$

where $h_1 = m_1/\mu'$, $h_2 = m_2/\mu'$. This is the theory we want to study. A depiction of its quiver diagram is shown in Figure 5.

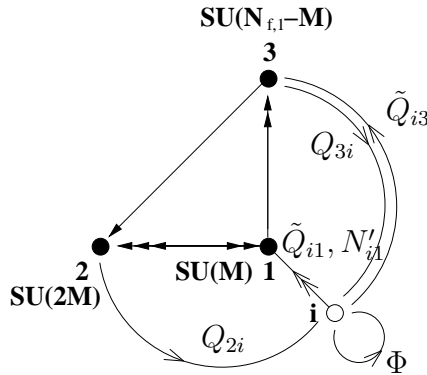


Figure 5: Quiver diagram for the dP_1 theory with flavors after dualization. Notice that the number of colors for nodes 1,2 and 3 are M , $2M$ and $N_{f,1} - M$. Since D7-branes are mixed after dualization, we represent them with a single white circle.

4.2 The local minimum

A detailed analysis of moduli and pseudomoduli in this theory and the computation of pseudomoduli masses is given in Appendix A.3. We present the results here. We first focus on the most symmetric choice of couplings $h = \lambda' = h_1 = h_2$ and $\mu = \mu_0$. A discussion for less symmetric couplings can be found in Appendix A.3. The moduli space of SUSY breaking minima is parametrized as follows

$$\begin{aligned} \tilde{Q}_{i3} &= \begin{pmatrix} \tilde{\varphi}_1 \\ 0 \end{pmatrix} & Q_{3i} &= (\varphi_1; 0) & \Phi &= \begin{pmatrix} 0 & 0 \\ 0 & \Phi_1 \end{pmatrix} \\ \tilde{Q}_{k1} &= \begin{pmatrix} 0 \\ y \end{pmatrix} & N'_{k1} &= \begin{pmatrix} 0 \\ z \end{pmatrix} & Q_{2i} &= \begin{pmatrix} 0 & x \\ 0 & x' \end{pmatrix} & M'_{12} &= \begin{pmatrix} xy \\ x'y \end{pmatrix} \end{aligned} \quad (4.8)$$

where $\varphi_1, \tilde{\varphi}_1$ are $N \times N$ matrices subject to $\tilde{\varphi}_1 \varphi_1 = \mu^2 \mathbf{1}_N$. The expectation values of all other fields vanish.

We now focus on the point of maximal unbroken global symmetry $\Phi_1 = x = x' = y = z = (\theta + \theta^*) = 0$. Computing the one loop effective potential we find that this critical point is a meta-stable minimum. Expanding around it, e.g. for the prototypical case of $N_{f,1} = 2M$, we get

$$\langle V_{eff}^{(1)} \rangle = const. + |h^4 \mu^2| \frac{(\log 4 - 1)}{16\pi^2} M^2 \left(2 |\delta\Phi_1|^2 + |\delta x|^2 + |\delta x'|^2 + |\delta y|^2 + |\delta z|^2 + |\mu^2|(\theta + \theta^*)^2 \right) + \dots \quad (4.9)$$

The striking similarity between these results and those of the extended model in Section 2.3 is explained in Appendix A.3. The longevity of this minimum is studied in Section 4.4.

Strictly speaking, the expectation value of Z_{12} is another pseudomodulus. This field does not appear in the classical superpotential so it is flat both at tree level and one-loop. Contrary to what happens for X_0 in the model of Section 2.2, motion along this direction does not take us closer to the SUSY vacua so we do not consider it poses an obvious danger. Nevertheless, it is in principle still possible that higher order corrections might render this field unstable, even modifying some of our conclusions. This is a very interesting direction for further research. In Section 6 we comment on another field, the saxion, with similar behavior.

An important final comment is that the existence of SUSY breaking local minima depends on some basic patterns of the theory. For instance, as explained in Section 3.2, the fact that in the original theory we have $N_{f,0} < N_c$, or the existence of singlets (of the strongly coupled gauge factor) which couple to the flavors. These features are present in general

quiver gauge theories of fractional branes at obstructed geometries, namely DSB fractional branes (in fact, it is easy to identify these features in all the examples of DSB fractional branes in [22]). We therefore expect that our conclusions for the dP_1 theory are of general validity for this whole class.

4.3 Behavior at large fields

We now explore whether, and if so where, this theory has a SUSY vacuum. As in previous theories, we expect to recover in the region of large fields the behavior of the theory without the extra flavors, determined in Section 3.3, namely a runaway for X_{12}, Y_{12} .

In fact, for generic non-zero vevs of fields $\Phi, \tilde{Q}_{k1}, Q_{2i}, N'_{k1}, M_{21}$ and M'_{21} the flavors of gauge factor 3 are massive and can be integrated out, leaving a pure $SU(N)$ SYM which triggers a non-perturbative superpotential. In more detail, there are a number of fields that contribute to the mass matrix of these flavors. Organizing the different fundamental flavors in a row vector q with entries $(Q_{3k}; X_{32})$ and the anti-fundamental flavors in a column vector $\tilde{q} = (\tilde{Q}_{i3}; X_{13}, Y_{13})$, the mass matrix is given by

$$m = \begin{pmatrix} h\Phi_{ki} & -h_1\tilde{Q}_{k1} & hN'_{k1} \\ -h_2Q_{2i} & hM_{21} & hM'_{21} \end{pmatrix} \quad (4.10)$$

This matrix can be used to integrate out massive fields. The low-energy effective dynamics is pure $SU(M)$ SYM with a dynamical scale Λ' obtained from matching

$$\Lambda'^{3M} = \frac{\det m}{\Lambda^{5M-2N_{f,1}}} \quad (4.11)$$

where Λ is the Landau pole scale of the IR free $SU(N_{f,1} - M)$ theory with $2M + N_{f,1}$ flavors.

The $SU(N_{f,1} - M)$ strong dynamics generates a non-perturbative superpotential. The complete superpotential, after integrating out the massive flavors and taking into account the non-perturbative dynamics, becomes

$$W = -h\mu^2 \text{tr} \Phi + h\mu_0 (M_{21}Y_{12} - M'_{21}X_{12}) + \lambda' Q_{2j} \tilde{Q}_{j1} X_{12} + (N_{f,1} - M) \left(\frac{\det m}{\Lambda^{5M-2N_{f,1}}} \right)^{\frac{1}{N_{f,1}-M}}$$

where one should recall that $\det m$ is a complicated function of the other fields in the theory.

Actually, it is easy to identify a particular direction in field space where the dynamics reduces to a runaway exactly like that of the un-flavored theory (3.3). Consider the equations of motion for $Q_{2j}, \tilde{Q}_{i1}, N'_{k1}$. It is straightforward to see that they can be satisfied by choosing $Q_{2j} = 0, \tilde{Q}_{i1} = 0, N'_{k1} = 0$. Along this direction we have

$$\det m = h^{N_{f,1}+2M} \det \Phi \det \mathcal{M} \quad (4.12)$$

with $\mathcal{M} = (M_{21}; M'_{21})$. So the effective theory along this solution is described by the superpotential

$$W = -h\mu^2 \text{tr } \Phi + h\mu_0 (M_{21}Y_{12} - M'_{21}X_{12}) + (N_{f,1} - M) \left(\frac{h^{N_{f,1}+2M} \det \Phi \det \mathcal{M}}{\Lambda^{5M-2N_{f,1}}} \right)^{\frac{1}{N_{f,1}-M}}$$

Using the equation of motion for Φ , we obtain

$$W = h\mu_0 (M_{21}Y_{12} - M'_{21}X_{12}) - M \left(\frac{\mu^{2N_{f,1}} \Lambda^{5M-2N_{f,1}}}{h^{2M} \det \mathcal{M}} \right)^{\frac{1}{M}} \quad (4.13)$$

This is essentially identical to (3.3), which described the dynamics of the un-flavored dP_1 theory.

4.4 Lifetime of meta-stable vacua

Comparing (2.22) and (4.13) and their derivations, we conclude that the discussion of the potential barrier height and lifetime of the local minimum in the flavored dP_1 theory is completely isomorphic to the one for the extension of SQCD with massless flavors of Section 2.3.

We translate from the SQCD model to dP_1 by identifying q_1 , \tilde{q}_1 and Σ_0 with Q_{3i} , \tilde{Q}_{i3} and $\mathcal{Y} = \begin{pmatrix} Y_{12} \\ X_{12} \end{pmatrix}$, respectively. The numbers of flavors and colors are $N_{f,0} = N_{f,1} = 2M$ and $N = M$. Finally, μ and μ_0 play the same role in both theories. Replacing in (2.32), we get the bounce action

$$S = \frac{1}{|h|^{26/3}} \frac{|\alpha_\mu|^8}{|\epsilon|^4} \quad (4.14)$$

which is independent of M . The interpretation of this result is identical to the one in Section 2.3 and we conclude that the meta-stable vacua can be made parametrically long-lived.

5. String theory construction

As already mentioned, the natural way to introduce fundamental flavors in D3-brane quiver gauge theories is by adding D7-branes passing through the singular points [55]. This introduces a new sector of open strings, stretching between the D3 and the D7-branes, leading to such flavors. In addition, there is a sector of open strings stretching among the D7-branes, but the corresponding fields have higher-dimensional support, and they behave as external

parameters from the viewpoint of the 4d gauge theory. In fact, due to the existence of superpotential couplings $X_{77'}\tilde{Q}_{7'3}Q_{37}$, they behave as masses for some of the flavors. Hence, D7-branes are the natural setup to introduce massive flavors in the string realization of quiver gauge theories.

To construct these configurations, we need efficient tools to classify interesting possibilities of D7-branes wrapped on non-compact 4-cycles on toric singularities, and to compute the open string 3-7 spectrum, and its interactions with the 3-3 sector. This study is carried out in Appendix B for a general toric singularity, and is applied in particular to the case of D7-branes in dP_1 .

In general it is not consistent to introduce just one kind of D7-brane in the configuration. D7-branes of the kind constructed in Appendix B carry non-trivial charge under RR 4-form fields localized at the singularity (obtained from higher-dimensional RR p -forms integrated over the compact homology cycles of the singularity), hence cancellation of such RR tadpoles requires combinations of such branes to be introduced simultaneously. Equivalently, the chiral spectrum of the 4d theory obtained from just one D7-brane has non-abelian anomalies. Hence, only combinations of D7-branes leading to an anomaly-free spectrum are allowed ¹⁵

For the case of the dP_1 theory, we can use the different D7-branes described in Appendix B.3 to obtain several ways to achieve this. Let us consider two simple classes of solutions (although others are possible as well): a) the four nodes in the original quiver get one fundamental flavor or b) only the three nodes in the final quiver get one fundamental flavor ¹⁶. The different possibilities of D7-branes to achieve this kind of spectrum are

$$\begin{aligned} \text{a)} \quad & (\Sigma''_{BC} \text{ or } \Sigma_{AB} \text{ or } \Sigma_{CD}) \oplus (\Sigma_{DB} \text{ or } \Sigma_{CA}) \oplus \Sigma_{AD} \oplus (\Sigma'_{DB} \text{ or } \Sigma'_{CA}) \\ \text{b)} \quad & (\Sigma''_{BC} \text{ or } \Sigma_{AB} \text{ or } \Sigma_{CD}) \oplus (\Sigma_{DB} \text{ or } \Sigma_{CA}) \oplus \Sigma_{BC} \end{aligned} \tag{5.1}$$

where \oplus denotes superposition, and ‘or’ denotes different alternative possibilities. The two classes are schematically shown in Figure Figure 6. The white circles represent the D7-branes.

¹⁵The equivalence of the statements is well-known for orbifolds [47, 46, 45]. It can be argued in general as follows: Given a set of D7-branes, consider the net number of fundamentals minus antifundamentals they introduce for a given D3-brane gauge group. This number corresponds to the charge of the D7-brane system under the RR field associated with the compact homology class corresponding to that D3-brane gauge factor. Since the homology classes of D3-brane gauge factors form a basis of the compact homology, anomaly cancellation is equivalent to zero compact homology charge for the D7-brane system.

¹⁶Here by original and final quivers we refer to the quivers for only regular D3-branes or only fractional branes respectively.

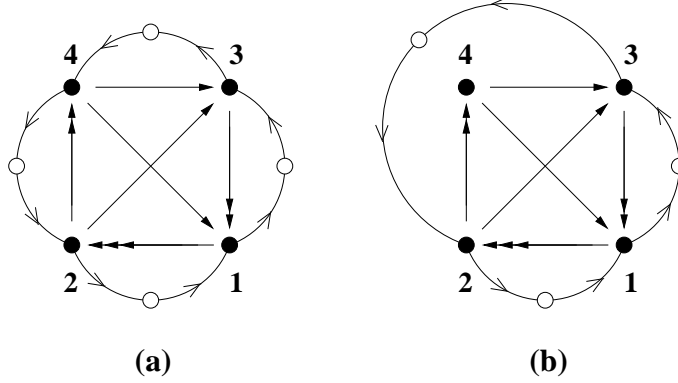


Figure 6: Two possible extended quivers corresponding to consistent sets of D7-branes in the dP_1 theory.

The above different possibilities lead, at the level of the quiver for the fractional branes, to the same spectrum of flavors, but differ in the superpotential couplings involving the latter.

For the sake of concreteness we will concentrate on one particular example, leading to the gauge theory studied in previous section, others can be worked out similarly. The configuration has $2M$ copies of the following set of D7-branes in class b , leading to $2M$ additional flavors for nodes 1, 2 and 3

$$\Sigma_{AB} \oplus \Sigma_{DB} \oplus \Sigma_{BC} \quad (5.2)$$

The 33-37-73 terms in the superpotential are

$$\tilde{Q}_{i2} X_{23} Q_{3i} \quad ; \quad \tilde{Q}_{j1} X_{12} Q_{2j} \quad ; \quad \tilde{Q}_{k3} X_{31} Q_{1k} \quad (5.3)$$

where Q, \tilde{Q} correspond to 37 and 73 states, respectively, namely fundamental and antifundamental flavors. As mentioned, this configuration of D7-branes reproduces the gauge theory studied in Section 4, with the 4d matter content in Figure 4 and the interactions in (4.1).

A last point that requires further discussion is the introduction of flavor masses. As already mentioned, they are controlled by vevs for fields in the D7-brane sector. Namely, the configuration contains couplings of the form $77'-7'3-37$, where 7 and 7' denote different D7-branes simultaneously present in the configuration. Since $77'$ fields have higher-dimensional support, their vevs are not dynamical fields from the 4d gauge theory viewpoint, but rather external parameters. Hence, as in several other familiar situations, the moduli space of the higher-dimensional theory provides the parameter space of the lower-dimensional one.

The higher-dimensional theory in this case has a complicated structure, since it involves gauge fields in the 77 open sector, hence propagating over an 8d space, and charged multiplets

from $77'$ open strings, in general propagating over 6d intersections. Without entering into a detailed general discussion, it suffices our purposes to consider one particular flat direction of this kind of theories. Namely, we consider the mesonic flat direction where a set of fields $\Phi_{7_1 7_2}, \Phi_{7_2 7_3}, \dots, \Phi_{7_k 7_1}$ acquire the same vev. From the viewpoint of the 4d gauge theory, this implies equal mass terms for all the $37_i, 7_i 3$ fields coupling to them.

To be more specific, let us consider our above example, namely the choice of D7-branes given in (5.2). From the discussion in Appendix B, such $77'$ sectors exist for pairs of D7-branes with a common letter in their label. Thus, we see that in our example we have a $77'$ open string bi-fundamental for each pair of D7-branes. The above mentioned mesonic flat direction corresponds to recombining the three intersecting cycles into a single smooth one, which is at a finite distance (controlled by the D7-brane field vevs) from the D3-branes at the singularity. This geometrical process can be modeled as follows. The initial configuration contains D7-branes on three holomorphic 4-cycles which intersect over a common holomorphic curve. In suitable local complex coordinates z_1, z_2, z_3 , the 4-cycles can be chosen to be $z_1 = 0, z_2 = 0, z_1 + z_2 = 0$, with the holomorphic curve thus given by $z_1 = z_2 = 0$ and spanned by z_3 . The complete D7-brane configuration is described by the equation $z_1 z_2 (z_1 + z_2) = 0$. Then the above mentioned mesonic branch corresponds to $z_1 z_2 (z_1 + z_2) = \epsilon$. The branes have recombined since the 4-cycle is now irreducible, and all D3-D7 open strings are massive because the 4-cycle does not pass through the origin.

6. Embedding into a duality cascade and breaking of baryonic $U(1)$

So far, we have studied in detail the 3+1 dimensional gauge theory that arises on fractional D3-branes on a toric singularity *in the absence of regular D3-branes*. We have added fundamental flavors by means of D7-branes. As we have argued, it is completely licit to study theories without regular D3-branes. On the other hand, these theories can be regarded as describing the IR bottom of a duality cascade [16]. In other words, embedding the theory in a duality cascade provides a specific UV completion. We now comment about a subtle point that should be contemplated in this case.

The gauge theory for a set of D3-branes and fractional branes at a singularity is not conformal and has a non-trivial RG flow. If the number of D3-branes N is much larger than the number of fractional branes M , the theory can be regarded as a small perturbation of the conformal theory. The general behavior is that, in analogy with the conifold [16], the theory undergoes cascades of Seiberg dualities along which the effective number of D3-branes is reduced as one moves to the IR. This fact is mapped to a radial dependence of

the 5-form flux in the corresponding gravity dual. For explicit examples of cascades, see [16, 17, 18, 19, 20]. Interestingly, when a small number of D7-branes is added, the effective number of fractional branes is also reduced along the RG flow (see for example [42]). This behavior translates into a radial dependence of the 3-form flux in the gravity dual [42]. It is straightforward to explicitly construct the duality cascade that appears when regular D3-branes are added to the flavored dP_1 theory that we have focused on in this paper. Since we are really interested in the bottom IR of it, we will skip doing so.

The number of D3, D5 and D7-branes in the UV can be appropriately chosen such that after a large number of dualizations we reach a point where the $N = M$ as shown in Figure 7¹⁷.

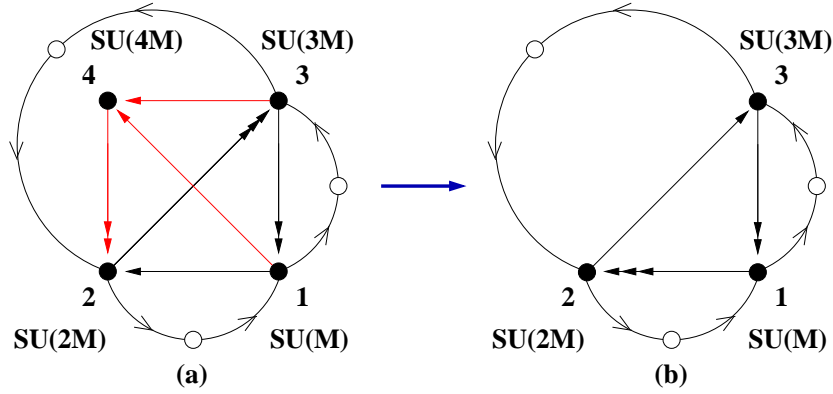


Figure 7: a) Quiver diagram for flavored dP_1 with M regular D3-branes and M D5-branes. b) The theory on the baryonic branch.

This theory, without the fundamental flavors, has been already investigated in [23], to which we refer the reader for details. We are interested in the situation in which the dynamics of node 4 becomes dominant. Since our choice of D7-branes is such that node 4 has no fundamental flavors, the discussion in [23] applies without changes. Since node 4 has an equal number of colors and flavors it has a quantum modified moduli space that is realized by adding the constraint $\det \mathcal{M} - B\tilde{B} = \Lambda_4^{8M}$ to the superpotential via a Lagrange multiplier. The mesons are combinations of the fields that we have indicated in red in Figure 7. Thorough analysis shows that the mesonic branch is completely lifted in this

¹⁷When doing a Seiberg duality transformation on a node, fundamental flavors of other nodes can appear as Seiberg mesons combining bifundamental and (anti)fundamental fields as in Section 4.1. Because of this, it seems possible to have a cascade in which the quiver in Figure 7.a is periodically repeated up to a change in the numbers of D3 and D5-branes and possibly permutations of the nodes.

theory. Along the baryonic branch, we obtain the flavored dP_1 theory of Section 4 (shown in Figure 7.b).

But we have to be cautious at this point. The global $U(1)$ baryonic symmetry of present in the original theory is spontaneously broken by the vevs of dibaryonic operators. As a result, the IR theory also contains a massless pseudo-scalar Goldstone boson (the "axion"). By supersymmetry, the axion falls into a massless $\mathcal{N} = 1$ chiral multiplet. Then, there will also be a massless scalar (the "saxion") and a Weyl fermion (the "axino"). The axion and the saxion are combined into the complex scalar of the chiral multiplet. The above argument is the generalization of the analysis in [40] of the conifold cascade.

While the axion is a Goldstone boson and remains massless, the flatness of the saxion can in principle be lifted by quantum corrections. At one-loop the saxion is decoupled from the flavored dP_1 sector and the computations of previous sections are not modified. Namely, the saxion remains massless at one-loop. It is not clear whether the saxion becomes unstable at higher loops or whether it can change any of our conclusions by coupling to the flavored dP_1 fields. We think that this is an interesting problem, important for the possible realization of gravitational duals of our theories, and which hence deserves further study.

7. Conclusions

In this paper we have studied the generalization of the ISS proposal to diverse gauge theories with massless flavors, including quiver gauge theories on fractional branes. Interestingly enough, the requirements of the ISS proposal (like SUSY breaking by the rank condition mechanism) suggest that the natural generalization for quiver gauge theories occurs for fractional branes in geometries with obstructed complex deformations. Although our detailed analysis has centered on concrete examples, our results have laid the grounds for more general analysis of this class of models.

Thus, it would be interesting to extend our computations to arbitrary number of extra flavors and of colors / fractional branes. Also, it would be interesting to analyze the introduction of flavors in other simple examples of DSB fractional branes (like the dP_2 or dP_3 theories).

The generalization of the ISS proposal for fractional branes in obstructed geometries leads effectively to a mechanism that allows to stay away from the runaway behavior of these configurations. This is an important development, that improves the possible application of these theories to dynamical SUSY breaking in String Theory model building (see [39] for a partial attempt).

It would be interesting to extend our discussion to other quiver gauge theories, for instance those arising in the presence of orientifold planes, so as to exploit the ISS mechanism for SO and Sp gauge theories. If the appropriate conditions are met, it is possible that addition of flavors can be used to escape the runaway behavior of models like that in [56].

Despite the progress made in this paper, we consider that it is important to be cautious about the fate of fields that remain decoupled from the rest of the theory and flat up to one-loop, such as Z_{12} in Section 4.2. This is an issue that needs to be understood in more detail. In particular it is worthy to understand whether they can become unstable and, if so, whether they modify any of our conclusions.

Another important open question concerns the realization of gravitational duals of the gauge theories at SUSY breaking minima. The generalization of the ISS mechanism to quiver gauge theories carried out in this paper is an important step in this direction. However, several other questions remain open. One of them is the saxion flat direction mentioned in Section 6. Another important point is that the large number of flavor branes requires the construction of supergravity solutions including their backreaction, which are very involved even in simple examples (for some discussions see e.g. [41, 42, 43, 44]).

We expect the fascinating physics of dynamical supersymmetry breaking and its realization in String Theory to continue triggering progress in these and other directions.

Acknowledgments

We thank M. Bertolini, I. García-Etxebarria, I. Klebanov, M. Kruczenski, L. McAllister, C. Nuñez, F. Saad, M. Wijnholt for useful discussions. We are specially grateful to K. Intriligator for reading the manuscript and useful comments. We thank the Department of Physics and Astronomy at UPenn, USA, and Centro Atómico Bariloche, Argentina, for hospitality during completion of this work. A.U. thanks M. González for encouragement and support.

A. Computation of pseudomoduli masses

The one-loop correction to the vacuum energy due to integrating out classically massive fields is

$$V_{eff}^{(1)} = \frac{1}{64\pi^2} \text{STr } \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} \equiv \frac{1}{64\pi^2} \left(\text{Tr } m_B^4 \log \frac{m_B^2}{\Lambda^2} - \text{Tr } m_F^4 \log \frac{m_F^2}{\Lambda^2} \right) \quad (\text{A.1})$$

where m_B^2 and m_F^2 are the classical squared masses for bosons and fermions as functions of the pseudomoduli vevs¹⁸.

In a theory of n chiral superfields Q^a with canonical classical Kähler potential, $K_{cal} = Q_a^\dagger Q^a$ and superpotential $W(Q_a)$, the scalar and fermion mass-squared matrices are given by

$$m_0^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & W^{\dagger abc} W_c \\ W_{abc} W^{\dagger c} & W_{ac} W^{\dagger cb} \end{pmatrix} \quad m_{1/2}^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & 0 \\ 0 & W_{ac} W^{\dagger cb} \end{pmatrix} \quad (\text{A.2})$$

where $W_c \equiv \partial W / \partial Q^c$, etc. The dimension of m_0^2 and $m_{1/2}^2$ is $2n \times 2n$. Supersymmetry breaking is encoded in the off-diagonal blocks of these matrices.

Typically, the effective potential for the pseudomoduli (A.1) generates masses for pseudomoduli when expanded around its critical points. These masses can be positive (the corresponding pseudomodulus is a stable direction) or negative (unstable direction). Sometimes, pseudomoduli remain massless at one-loop. Since they are not Goldstone bosons, their masses are not protected from perturbative corrections and it is therefore expected that their flatness is lifted at some higher order.

A.1 Massless flavored SQCD

The superpotential for this theory is

$$W = h \text{Tr } q_0 \Phi_{00} \tilde{q}_0 + \text{Tr } q_0 \Phi_{01} \tilde{q}_1 + \text{Tr } q_1 \Phi_{10} \tilde{q}_0 + \text{Tr } q_1 \Phi_{11} \tilde{q}_1 - h \mu^2 \text{Tr } \Phi_{11} \quad (\text{A.3})$$

The F-term for Φ_{11} breaks supersymmetry due to the rank condition. The classical minima of this potential are obtained by saturating this F-term as much as possible. There is a moduli space of field configurations satisfying this with $V_{min} = (N_{f,1} - N) |h^2 \mu^4|$. In particular, for any choice of vevs for q_1, \tilde{q}_1 of the form

$$q_1 = (\varphi_1; 0) \quad \tilde{q}_1 = \begin{pmatrix} \tilde{\varphi}_1 \\ 0 \end{pmatrix} \quad (\text{A.4})$$

with $\tilde{\varphi}_1 \varphi_1 = \mu^2 \mathbf{1}_N$. Here $\tilde{\varphi}_1, \varphi_1$ are $N \times N$ blocks.

In order to make the F-terms of Φ_{01}, Φ_{10} vanish, q_0 and \tilde{q}_0 must vanish. In addition, to make the F-terms of q_1 and \tilde{q}_1 vanish, the vevs of $\Phi_{10}, \Phi_{01}, \Phi_{11}$ must be of the form

$$\Phi_{01} = (0; Y) \quad \Phi_{10} = \begin{pmatrix} 0 \\ \tilde{Y} \end{pmatrix} \quad \Phi_{11} = \begin{pmatrix} 0 & 0 \\ 0 & X_1 \end{pmatrix} \quad (\text{A.5})$$

¹⁸The ultraviolet cutoff Λ can be absorbed in the renormalization of the couplings that appear in the tree-level vacuum energy.

where Y , \tilde{Y} , X_1 are $N_{f,0} \times (N_{f,1} - N)$, $(N_{f,1} - N) \times N_{f,0}$ and $(N_{f,1} - N) \times (N_{f,1} - N)$ blocks, respectively. Finally, the vev for $\Phi_{00} = X_0$ is arbitrary.

We can use $SU(N_{f,0})$ global symmetry transformations to make X_0 diagonal. Furthermore, X_1 can be diagonalized by means of $SU(N - N_{f,1})$ transformations.

We now expand fields in fluctuations around arbitrary expectation values of the the pseudomoduli

$$\begin{aligned}
q_0 &= \delta\rho_0 & \tilde{q}_0 &= \delta\tilde{\rho}_0 & q_1 &= (\mu e^\theta \mathbf{1}_N + \delta\chi; \delta\rho_1) & \tilde{q}_1 &= \begin{pmatrix} \mu e^{-\theta} \mathbf{1}_N + \delta\tilde{\chi} \\ \delta\tilde{\rho}_1 \end{pmatrix} \\
\Phi_{00} &= X_0 + \delta\Phi_0 & \Phi_{01} &= (\delta W; Y + \delta Y) \\
\Phi_{1,0} &= \begin{pmatrix} \delta\tilde{W} \\ \tilde{Y} + \delta\tilde{Y} \end{pmatrix} & \Phi_{11} &= \begin{pmatrix} \delta Y_1 & \delta Z_1 \\ \delta\tilde{Z}_1 & X_1 \mathbf{1}_{N_f - N} + \delta\Phi_1 \end{pmatrix}
\end{aligned} \tag{A.6}$$

We must now expand the classical superpotential to quadratic order in the fluctuations, except for the terms involving $\delta\Phi_1$, for which we must allow cubic terms. The reason is that $\delta\Phi_1$ is the only field with non-vanishing F-term in the vacuum, so it leads to a contribution to the scalar mass matrix involving third derivatives of the terms in which $\delta\Phi_1$ appears. The result is

$$\begin{aligned}
W &= h \text{Tr} \left[\delta\rho_1 (X_1 + \delta\Phi_1) \delta\tilde{\rho}_1 - \mu^2 (X_1 + \delta\Phi_1) + \mu e^\theta \delta Z_1 \delta\tilde{\rho}_1 + \mu e^{-\theta} \delta\rho_1 \delta\tilde{Z}_1 + \right. \\
&\quad + \mu e^\theta \delta Y_1 \delta\tilde{\chi}_1 + \mu e^{-\theta} \delta\chi_1 \delta\tilde{Y}_1 + \mu e^{-\theta} \delta\rho_0 \delta W + \mu e^\theta \delta\tilde{W} \delta\tilde{\rho}_0 + \\
&\quad \left. + \delta\rho_0 X_0 \delta\tilde{\rho}_0 + \delta\rho_0 Y \delta\tilde{\rho}_1 + \delta\rho_1 \tilde{Y} \delta\tilde{\rho}_0 \right]
\end{aligned} \tag{A.7}$$

As in SQCD with massive flavors [27], off diagonal elements of $\delta\Phi_1$ do not enter in the mass matrix. The same thing happens for $\delta\Phi_0$. This is natural since they are Goldstone bosons of the broken global symmetries, which obviously remain exactly massless (to any order).

The fields δY_1 , $\delta\chi_1$, $\delta\tilde{\chi}_1$ are decoupled from the SUSY breaking sector (i.e. from fields with a non-supersymmetric mass matrix). As a result, they have a supersymmetric mass matrix and do not contribute to the supertrace.

The main difference with respect to the case studied in [27] is given by the generically rectangular matrices Y and \tilde{Y} coupling fluctuations. Nevertheless, in analogy with [27], one can use symmetries of the system to show that the lagrangian of pseudomoduli masses is given by a sum of terms $\text{tr } M^\dagger M$, where M denotes the different matrix pseudomoduli (see (A.9) later). The coefficients of such terms can be computed by taking a simple ansatz for

the corresponding pseudomodulus. That is, we take Y to be formed by a diagonal block $\text{diag}(Y_1, \dots, Y_{\tilde{N}})$, where $\tilde{N} = \min(N_{f,0}, N_{f,1} - N)$, and an appropriately located block of zeroes (i.e. either additional rows or columns) to complete the $N_{f,0} \times (N_{f,1} - N)$ dimensions. We take the matrix \tilde{Y} to have an analogous block-diagonal form.

Now, the interpretation of (A.7) is clear, relating it to O’Raighfertaigh-like (OR) models of the kind used in [27]. For $(N_{f,1} - N) < N_{f,0}$ it corresponds to the sum of $(N_{f,1} - N)$ copies of an OR model involving all terms in (A.7). For $N_{f,0} < (N_{f,1} - N)$ we have $N_{f,0}$ copies of an OR model with all the terms in (A.7), plus $(N_{f,1} - N - N_{f,0})$ copies of an OR model in which all the terms in (A.7) except the last two are present (this latter OR model effectively corresponds to that used in [27]).

We then compute m_0^2 and $m_{1/2}^2$ using (A.2) and plug the result into (A.1) to obtain the one-loop effective potential. The full expression for $V_{eff}^{(1)}$ is very complicated. For illustrative purposes we present it for fixed values $Y = \tilde{Y} = \theta = 0$

$$\begin{aligned} V_{eff}^{(1)}|_{Y, \tilde{Y}, \theta=0}(X_1, X_0) = & \frac{h^4 N(N_f - N)}{128\pi^2} \left[4(\mu^2 + X_1^2)^2 \log\left(\frac{h^2(\mu^2 + X_1^2)}{\Lambda}\right) \right. \\ & + \left(3\mu^2 + X_1^2 - \sqrt{\mu^4 + 6\mu^2 X_1^2 + X_1^4}\right)^2 \log\left(\frac{h^2(3\mu^2 + X_1^2 - \sqrt{\mu^4 + 6\mu^2 X_1^2 + X_1^4})}{2\Lambda}\right) \\ & + \left(3\mu^2 + X_1^2 + \sqrt{\mu^4 + 6\mu^2 X_1^2 + X_1^4}\right)^2 \log\left(\frac{h^2(3\mu^2 + X_1^2 + \sqrt{\mu^4 + 6\mu^2 X_1^2 + X_1^4})}{2\Lambda}\right) \\ & + 2\left(2\mu^2 + X_1\left(X_1 - \sqrt{4\mu^2 + X_1^2}\right)\right)^2 \log\left(\frac{h^2(2\mu^2 + X_1(X_1 - \sqrt{4\mu^2 + X_1^2}))}{2\Lambda}\right) \\ & \left. - 2\left(2\mu^2 + X_1\left(X_1 + \sqrt{4\mu^2 + X_1^2}\right)\right)^2 \log\left(\frac{h^2(2\mu^2 + X_1(X_1 + \sqrt{4\mu^2 + X_1^2}))}{2\Lambda}\right) \right] \quad (\text{A.8}) \end{aligned}$$

which does not depend on X_0 at all. In order to keep the above expression compact, we have omitted absolute values.

The full one-loop effective potential has a critical point at the vacua of maximal unbroken global symmetry, which correspond to $X_0 = X_1 = Y = \tilde{Y} = 0$ and $\varphi_1 = \tilde{\varphi}_1 = \mu$ (up to unbroken flavor rotations).

Expanding the effective potential around these pseudomoduli vevs, we obtain

$$\langle V_{eff}^{(1)} \rangle = \text{const.} + |h^4 \mu^2| \frac{(\log 4 - 1)}{16\pi^2} N \left[(N_f - N) \left(2|\delta\Phi_1|^2 + |\mu^2|(\theta + \theta^*)^2 \right) + |\delta Y|^2 + |\delta\tilde{Y}|^2 \right] + \dots \quad (\text{A.9})$$

where $|\delta Y|^2$ and $|\delta\tilde{Y}|^2$ should be understood as the squared norms of \tilde{N} -dimensional complex vectors. Taking the non-trivial diagonal blocks of Y and \tilde{Y} to be proportional to $\mathbf{1}_{\tilde{N}}$, (A.9) simplifies to

$$\langle V_{eff}^{(1)} \rangle = const. + |h^4 \mu^2| \frac{(\log 4 - 1)}{16\pi^2} N \left[(N_f - N) \left(2 |\delta\Phi_1|^2 + |\mu^2|(\theta + \theta^*)^2 \right) + \tilde{N} \left(|\delta Y|^2 + |\delta \tilde{Y}|^2 \right) \right] + \dots \quad (\text{A.10})$$

From A.9, we see that $\delta\Phi_0$ remains massless at one-loop, i.e. X_0 remains a flat direction. An heuristic, although not rigorous, way to understand why this happens is to look at (A.7) and notice that for $Y = \tilde{Y} = 0$, X_0 decouples from the SUSY breaking sector and thus it does not contribute to the supertrace, disappearing from $V_{eff}^{(1)}$. Outside the critical point, for $Y, \tilde{Y} \neq 0$, X_0 couples to the SUSY breaking sector and appears in the effective potential, but it becomes an unstable direction as shown in Figure 8.

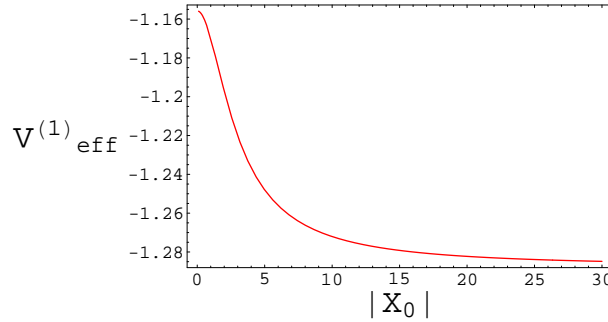


Figure 8: The one-loop effective potential in arbitrary units as a function of Φ_0 for $Y \neq 0$ and $\Phi_1 = \tilde{Y} = \theta = 0$.

In principle, it is still possible that $\delta\Phi_0$ becomes massive at higher loops, producing a meta-stable minimum (probably with a much smaller potential barrier) at small expectation values for the fields. We do not consider this possibility but explore a different direction in Section 2.3. The **most economical** way to lift this flat direction classically is by considering a modified toy model, with the addition to the electric theory of a neutral field Σ_0 with cubic coupling to massless flavors.

A.2 Extended model

The pseudomoduli in this case are identical to those in (A.4) and (A.5) with the exception that the coupling (2.20) ‘freezes’ the expectation value of X_0 to zero in the magnetic theory as discussed in Section 2.3. This occurs already at the classical level so X_0 is not a pseudomoduli in this theory.

Repeating the derivation in the previous section, the expansion of the superpotential to second order in fluctuations (and to third order in terms involving $\delta\Phi_1$ which has a non-vanishing F-term) is

$$W = h \text{Tr} \left[\delta\rho_1 (X_1 + \delta\Phi_1) \delta\tilde{\rho}_1 - \mu^2 (X_1 + \delta\Phi_1) + \mu e^\theta \delta Z_1 \delta\tilde{\rho}_1 + \mu e^{-\theta} \delta\rho_1 \delta\tilde{Z}_1 + \right. \\ \left. + \mu e^{-\theta} \delta\rho_0 \delta W + \mu e^\theta \delta\tilde{W} \delta\tilde{\rho}_0 + \delta\rho_0 Y \delta\tilde{\rho}_1 + \delta\rho_1 \tilde{Y} \delta\tilde{\rho}_0 \right] \quad (\text{A.11})$$

where we have already dropped fields that do not couple to the SUSY breaking sector. The effective potential has a critical point at the vacua of maximal unbroken global symmetry given, up to unbroken flavor rotations, by $X_1 = Y = \tilde{Y} = 0$ and $\varphi_1 = \tilde{\varphi}_1 = \mu$.

As in the previous section, we can split the lagrangian for the fluctuations into a sum of simple OR modes.

Expanding $V_{eff}^{(1)}$ around these vacua we obtain

$$\langle V_{eff}^{(1)} \rangle = const. + |h^4 \mu^2| \frac{(\log 4 - 1)}{16\pi^2} N \left[(N_f - N) \left(2 |\delta\Phi_1|^2 + |\mu^2|(\theta + \theta^*)^2 \right) + |\delta Y|^2 + |\delta\tilde{Y}|^2 \right] + \dots \quad (\text{A.12})$$

i.e. all pseudomoduli are lifted at one-loop and we have a SUSY breaking, meta-stable minimum at $X_1 = Y = \tilde{Y} = 0$ and $\varphi_1 = \tilde{\varphi}_1 = \mu$. Once again, taking the diagonal blocks of Y and \tilde{Y} to be proportional to $\mathbf{1}_{\tilde{N}}$, (A.12) reduces to

$$\langle V_{eff}^{(1)} \rangle = const. + |h^4 \mu^2| \frac{(\log 4 - 1)}{16\pi^2} N \left[(N_f - N) \left(2 |\delta\Phi_1|^2 + |\mu^2|(\theta + \theta^*)^2 \right) + \tilde{N} \left(|\delta Y|^2 + |\delta\tilde{Y}|^2 \right) \right] + \dots \quad (\text{A.13})$$

A.3 Flavored dP_1

The quiver diagram for this model is shown in Figure 5. The superpotential is given by (4.7) and corresponds to

$$W = h \Phi_{ki} \tilde{Q}_{i3} Q_{3k} - h \mu^2 \text{tr} \Phi + h \mu_0 (M_{21} Y_{12} - M'_{21} X_{12}) + \\ + h (M_{21} X_{13} X_{32} + M'_{21} Y_{13} X_{32} + N'_{k1} Y_{13} Q_{3k}) + \\ + \lambda' Q_{2j} \tilde{Q}_{j1} X_{12} - h_1 \tilde{Q}_{k1} X_{13} Q_{3k} - h_2 Q_{2i} \tilde{Q}_{i3} X_{32} \quad (\text{A.14})$$

The first step is to parametrize the SUSY breaking vacua and to identify the pseudo-moduli. Given the complicated structure of the theory this task is slightly more involved

than in previous examples. There are 15 chiral fields in this theory. We simply impose the minimization of their F-terms (actually all F-terms, except for the one of Φ , vanish) in a convenient order.

We start with

$$\frac{\partial W}{\partial \Phi} = 0 \quad \rightarrow \quad \tilde{Q}_{i3} Q_{3k} = \mu^2 \mathbf{1}_{\mathbf{N}_{f,1}} \quad (\text{A.15})$$

This cannot be saturated and breaks SUSY by the rank condition. This F-term contribution is minimized by choosing

$$\tilde{Q}_{i3} = \begin{pmatrix} \tilde{\varphi}_1 \\ 0 \end{pmatrix} \quad ; \quad Q_{3i} = (\varphi_1; 0) \quad (\text{A.16})$$

where $\varphi_1, \tilde{\varphi}_1$ are $N \times N$ matrices and $\tilde{\varphi}_1 \varphi_1 = \mu^2 \mathbf{1}_{\mathbf{N}}$.

Now we impose

$$\begin{aligned} \partial_{Y_{12}} W &= 0 \quad \rightarrow \quad M_{21} = 0 \\ \partial_{N'_{k1}} W &= 0 \quad \rightarrow \quad Y_{13} Q_{3k} = 0 \quad \rightarrow \quad Y_{13} = 0 \\ \partial_{M'_{21}} W &= 0 \quad \rightarrow \quad Y_{13} X_{32} - \mu_0 X_{12} = 0 \quad \rightarrow \quad X_{12} = 0 \end{aligned} \quad (\text{A.17})$$

So far, the results are independent of the presence of couplings h_1 and h_2 . From now on, we make explicit use of them (in their absence, vevs are less constrained, leading to more pseudomoduli). Let us continue by imposing

$$\begin{aligned} \partial_{\tilde{Q}_{i1}} W &= 0 \quad \rightarrow \quad \lambda' X_{12} Q_{2i} - h_1 X_{13} Q_{3i} = 0 \quad \rightarrow \quad X_{13} = 0 \\ \partial_{Q_{2i}} W &= 0 \quad \rightarrow \quad \lambda' \tilde{Q}_{i1} X_{12} - h_2 \tilde{Q}_{i3} X_{32} = 0 \quad \rightarrow \quad X_{32} = 0 \\ \partial_{M_{21}} W &= 0 \quad \rightarrow \quad X_{13} X_{32} + \mu_0 Y_{12} = 0 \quad \rightarrow \quad Y_{12} = 0 \end{aligned} \quad (\text{A.18})$$

With this, we are ready to get the expression for Φ . We have

$$\begin{aligned} \partial_{\tilde{Q}_{i3}} W &= 0 \quad \rightarrow \quad h Q_{3k} \Phi_{ki} - h_2 X_{32} Q_{2i} = 0 \quad \rightarrow \quad Q_{3k} \Phi_{ki} = 0 \\ \partial_{Q_{3k}} W &= 0 \quad \rightarrow \quad h \Phi_{ki} \tilde{Q}_{i3} - h_1 \tilde{Q}_{k1} X_{13} + N'_{k1} Y_{13} = 0 \quad \rightarrow \quad \Phi_{ki} \tilde{Q}_{i3} = 0 \end{aligned} \quad (\text{A.19})$$

From these two conditions Φ has the structure

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_1 \end{pmatrix} \quad (\text{A.20})$$

Notice that the fact that h_1 and h_2 are non-vanishing is crucial for getting this. Continuing, we have

$$\begin{aligned}
\partial_{X_{13}} W = 0 & \rightarrow hX_{32}M_{21} - h_1Q_{3k}\tilde{Q}_{k1} = 0 \rightarrow \tilde{Q}_{k1} = \begin{pmatrix} 0 \\ y \end{pmatrix} \\
\partial_{Y_{13}} W = 0 & \rightarrow X_{32}M'_{21} + Q_{3k}N'_{k1} = 0 \rightarrow N'_{k1} = \begin{pmatrix} 0 \\ z \end{pmatrix} \\
\partial_{X_{32}} W = 0 & \rightarrow hM_{21}X_{13} + hM'_{21}Y_{13} - h_2Q_{2i}\tilde{Q}_{i3} = 0 \rightarrow Q_{2i} = \begin{pmatrix} 0 & x \\ 0 & x' \end{pmatrix} \\
\partial_{X_{12}} W = 0 & \rightarrow -h\mu_0M'_{21} + \lambda'Q_{2j}\tilde{Q}_{j1} = 0 \rightarrow M'_{12} = \frac{\lambda'}{h\mu_0} \begin{pmatrix} xy \\ x'y \end{pmatrix} \quad (\text{A.21})
\end{aligned}$$

So far we have minimized 14 of the F-term contributions. We are only left with Z_{12} which does not appear in the superpotential in (A.17) and thus its contribution trivially vanishes.

Notice that only the vev of M'_{12} depends on superpotential couplings. Furthermore, none of the vevs depend on h_1 or h_2 . These facts will be important when studying the effective potential for general values of the couplings.

Expanding the superpotential to quadratic order in fluctuations, except for terms involving $\delta\Phi_{11}$ (the only field with a non-zero F-term in the vacuum) which have to be expanded to cubic order, we obtain

$$\begin{aligned}
W = \text{Tr} \left[-h\mu^2\Phi_1 + h\Phi_1\delta Q_{3,2}\delta\tilde{Q}_{3,2} + h\mu e^\theta\delta Y_{13}\delta N_{1,1} - h_1\mu e^\theta\delta X_{13}\delta\tilde{Q}_{1,1} - h_1y\delta X_{13}\delta Q_{3,2} + hz\delta Y_{13}\delta Q_{3,2} \right. \\
+ \frac{xy\lambda'}{\mu_0}\delta Y_{13}\delta X_{32,1} - h_2\mu e^{-\theta}\delta Q_{2,11}\delta X_{32,1} - h_2x\delta\tilde{Q}_{3,2}\delta X_{32,1} + \frac{x'y\lambda'}{\mu_0}\delta Y_{13}\delta X_{32,2} - h_2\mu e^{-\theta}\delta Q_{2,21}\delta X_{32,2} \\
\left. - h_2x'\delta\tilde{Q}_{3,2}\delta X_{32,2} + h\mu e^\theta\delta\tilde{Q}_{3,2}\delta\Phi_{01} + h\mu e^{-\theta}\delta Q_{3,2}\delta\Phi_{10} - h\mu^2\delta\Phi_{11} + h\delta Q_{3,2}\delta\tilde{Q}_{3,2}\delta\Phi_{11} \right] \quad (\text{A.22})
\end{aligned}$$

where we have only kept those terms that couple to supersymmetry breaking fields and thus give a non-vanishing contribution to the supertrace when integrated out. We have used the obvious notation for fluctuations, including additional subindices to indicate matrix subblocks (that reduce to matrix entries for $M = 1$). In analogy with (A.6), θ is defined by

$$\tilde{Q}_{i3} = \begin{pmatrix} \mu e^\theta \mathbf{1}_N + \delta\tilde{Q}_{3,1} \\ \tilde{Q}_{3,2} \end{pmatrix} \quad ; \quad Q_{3i} = (\mu e^{-\theta} \mathbf{1}_N + \delta Q_{3,1}; \delta Q_{3,2}) \quad (\text{A.23})$$

Let us consider in a little more detail those fluctuations that are absent from (A.22) because they have a supersymmetric matrix. Fluctuations of classically massive fields δM_{21} ,

$\delta M'_{21}$, δX_{12} and δY_{12} are naturally expected not to contribute to the effective potential. Also $\delta Q_{3,1}$, $\delta \tilde{Q}_{3,1}$ and $\delta \Phi_{11}$ do not contribute, in complete analogy with similar fields in SUSY QCD with massive flavors. The only new ingredient is the fact that $\delta Q_{2,12}$ and $\delta Q_{2,22}$ disappear.

Until now, we have kept our discussion completely general. In order to compute the effective potential we have to diagonalize the mass matrices (A.2). Doing this analytically is intractable for generic values of the superpotential couplings. We now focus on the most symmetric choice of couplings, i.e. $h = \lambda' = h_1 = h_2$ and $\mu = \mu_0$.

As in the previous examples, the lagrangian for the fluctuations splits into a sum of OR models. We now focus in the case $N_{f,1} = 2M$. Since $N_{f,0} = 2M$, this implies $N = M$. The effective potential has a minimum at $\Phi_1 = x = x' = y = z = (\theta + \theta^*) = 0$. Expanding around it, we have

$$\langle V_{eff}^{(1)} \rangle = const. + |h^4 \mu^2| \frac{(\log 4 - 1)}{16\pi^2} M^2 \left(2 |\delta \Phi_1|^2 + |\delta x|^2 + |\delta x'|^2 + |\delta y|^2 + |\delta z|^2 + |\mu^2|(\theta + \theta^*)^2 \right) + \dots \quad (\text{A.24})$$

This result is remarkably similar to the one for the extension of SQCD with massless flavors of Appendix A.2. This is due to the close similarity between (A.11) and (A.22). In fact, it is possible to identify analogous fields in both models

Extended massless SQCD	Flavored dP_1	
Φ_{00}	M_{21}, M'_{21}	
Φ_{01}	N'_{i1}	similar mesons, N_{i1} , become massive
Φ_{10}	N_{2i}	they are massive
Φ_{11}	Φ	

(A.25)

The analogue of Φ_{10} in flavored dP_1 are the flavors N_{2i} coming out of node 2. These fields are massive and do not appear in the final theory. Despite this, Q_2 takes a vev of similar form to that of Φ_{10} . Similarly, \tilde{Q}_1 is not analogous to Φ_{01} but its vevs have the same structure.

Extending the analysis to different couplings

We have just considered a very symmetric situation in which all dimensionless couplings in the superpotential are identical. It is natural to wonder whether meta-stable minima still exist in the case in which the couplings are different. In order to study this question we

consider the case in which $h_1 = h_2 = \lambda'$, but we allow h_1 to be different from h . In this case we cannot treat the problem analytically anymore and we proceed numerically.

As before, $V_{eff}^{(1)}$ has a critical point for $\Phi_1 = x = x' = y = z = (\theta + \theta^*) = 0$. Expanding around it we conclude that the mass of the x , x' and $\theta + \theta^*$ fluctuations depend only on h while masses of y and z fluctuations seem to be equal and depend on both h and h_1 . Near the critical point, we have¹⁹

$$\begin{aligned} \langle V_{eff}^{(1)} \rangle = & const. + |h^4 \mu^2| \frac{(\log 4 - 1)}{16\pi^2} (2 |\delta\Phi_1|^2 + |\delta x|^2 + |\delta x'|^2 + |\mu^2|(\theta + \theta^*)^2) + \\ & + m_y^2 |\delta y|^2 + m_z^2 |\delta z|^2 + \dots \end{aligned} \quad (\text{A.26})$$

Figure 9 shows the behavior of $m_y^2 = m_z^2$ as a function of h_1/h .

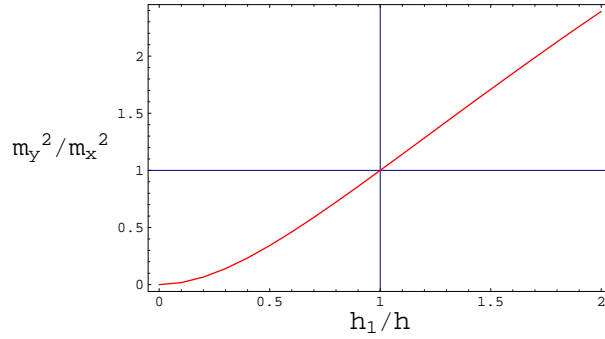


Figure 9: The value of m_y^2/m_x^2 as a function of h_1/h . We indicate the point of symmetric couplings $h_1 = h$.

We can consider more involved situations in which all couplings are different. For small variations of the couplings, the existence of a meta-stable minimum is guaranteed. This follows because the moduli space of the theory for equal couplings is compact, and the small variation of couplings can be regarded as a small potential on it. We have moreover performed a numerical analysis in some directions in coupling space. Our analysis seems to indicate that the existence of meta-stable, SUSY breaking minima where all pseudomoduli get positive masses at one-loop and are consequently lifted is robust with respect to variations of the couplings.

¹⁹Although we have computed the effective potential numerically for specific values of h , h_1 and μ , we provide an analytical expression in (A.26). With our analysis, we can only say that this expression is correct to a high numerical accuracy.

B. Flavor D7-branes for D3-branes at singularities from dimers

The introduction of flavors in gauge theories realized on systems of D3-branes at singularities is naturally achieved by incorporating D7-branes. Such systems have been considered in geometries related to flat space or orbifolds in e.g. [53, 54, 51, 41, 55]. However, for D3-branes at non-orbifold singularities, it is non-trivial to obtain the D3-D7 open string spectrum and interactions (see e.g. [51, 42] for discussion in some simple examples). In this appendix we introduce techniques to construct and characterize a simple class of D7-branes wrapped on holomorphic 4-cycles in systems of D3-branes at general toric Calabi-Yau three-fold singularities. Our characterization is based on the description of these systems in terms of dimer diagrams (or brane tilings) [30, 31, 32, 33, 34]. More specifically, our main tool is the Riemann surface Σ in the mirror configuration, studied in [33] (and whose skeleton is the web diagram).

Before entering the discussion, a comment is in order. In this appendix (and pieces of the main text using its results), each gauge factor arising on the D3-brane gauge theory is loosely referred to as a ‘D3-brane’.

B.1 General lessons from dP_0

Let us start with a heuristic argument. In systems of D3-branes at singularities, supersymmetric flavor D7-branes wrap holomorphic non-compact 4-cycles. A set of these (the so-called toric divisors) are associated with external points in the toric diagram, or non-compact faces in web diagrams, spanned by adjacent external legs. To be concrete, the web diagram of the complex cone over dP_0 , namely the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold singularity, is shown in Figure 10, along with the three basic non-compact 4-cycles. They correspond to the three 4-cycles defined by $z_i = 0$, $i = 1, 2, 3$, where z_i denote complex coordinates of \mathbb{C}^3 , which descend to the orbifold space.

This orbifold example already shows a crucial subtlety. Starting with D7-branes wrapped on a 4-cycle e.g. $z_1 = 0$ of the parent \mathbb{C}^3 , there are three possible choices of Chan-Paton factor in quotienting by \mathbb{Z}_3 , given by the three roots of unity. This shows that for each 4-cycle there are three different discrete choices that define a D7-brane in the orbifold geometry. In fact, different choices of Chan-Paton factors lead to different D3-D7 spectra, etc. Hence it is important to characterize this subtle feature already in the quotient space. By doing so we will be able to generalize to other examples, including non-orbifold ones, where the notion of Chan-Paton factor is actually not so familiar.

The structure of D7-branes, including discrete multiplicities, turns out to be very simple in the mirror geometry. Recall that in the mirror geometry a prominent role is played by the

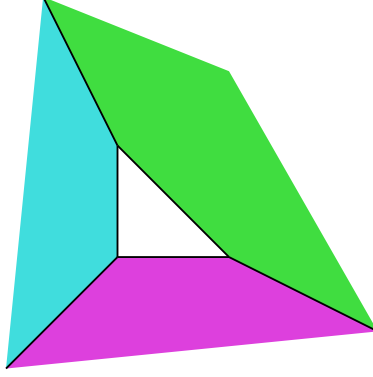


Figure 10: Three basic 4-cycles in the dP_0 theory.

(punctured) Riemann surface Σ whose skeleton is the web diagram. This Riemann surface can be obtained by considering the zig-zag paths of the dimer diagram, which provide a tiling of Σ where faces correspond to zig-zag paths, and their adjacency can be read from the dimer diagram. In Figure 11 we give the dimer diagram and zig-zag paths for the dP_0 theory. The mirror Riemann surface Σ is shown in Figure 12.

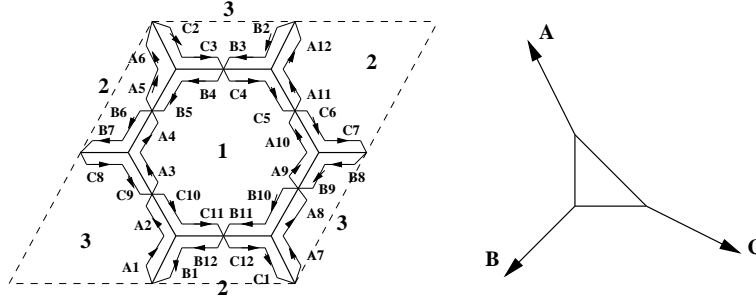


Figure 11: Dimer and zig-zag paths in the dP_0 theory.

In this mirror picture, the different D3-brane gauge factors arise from D6-branes on compact 3-cycles, which are encoded in non-trivial compact 1-cycles in Σ . Amusingly, these are zig-zag paths of the tiling of Σ . Moreover, the bifundamental chiral multiplets arise from intersections of these 3-cycles, and their superpotential couplings arise from disks bounded by the 1-cycles. The 1-cycles associated with the three D3-brane gauge factors are shown in Figure 14. Notice the geometric \mathbb{Z}_3 symmetry (mirror to the quantum \mathbb{Z}_3 symmetry of the orbifold) exchanging them. It is easy to check the intersection numbers of the 1-cycles

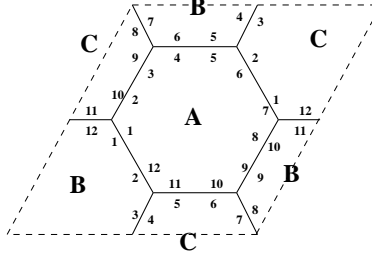


Figure 12: Mirror Riemann surface for the dP_0 theory. Sides in this picture are identified so that the topology of Σ is that of a (punctured) genus 1 Riemann surface. Punctures are located in the middle of the faces in the picture. The fact that the tiling of Σ is similar to the original brane tiling is a property of del Pezzo theories, and not valid for a general singularity.

and recover the quiver diagram for the dP_0 theory, shown in Figure 13. Also, the cubic superpotential couplings can be obtained from triangles bounded by pieces of 1-cycles in Σ , see Figure 15.

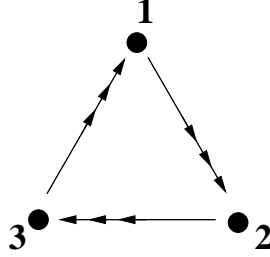


Figure 13: Quiver for the dP_0 theory.

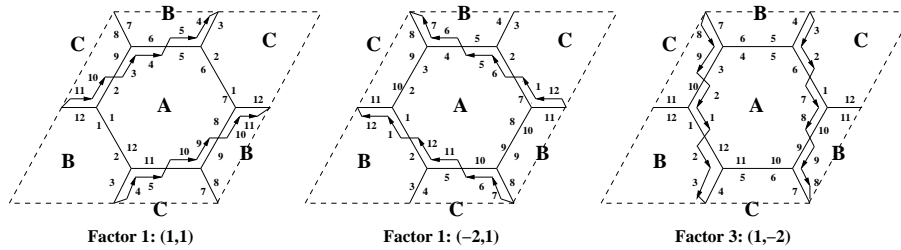


Figure 14: The 1-cycles for the three D3-brane gauge factors in the dP_0 theory.

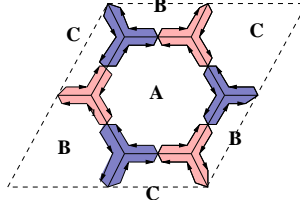


Figure 15: Disks corresponding to the cubic couplings in the superpotential of the dP_0 theory.

It is now easy to describe the D7-branes in this picture. In the mirror geometry, they correspond to D6-branes wrapped on non-compact 3-cycles. These correspond to non-compact 1-cycles in Σ , which come from infinity at one puncture and go to infinity at another puncture²⁰ (so in the web diagram they are naturally associated with non-compact faces spanned by the corresponding legs). An intuitive picture of the 1-cycles in Σ associated with D3 and D7-branes is shown in Figure 16.

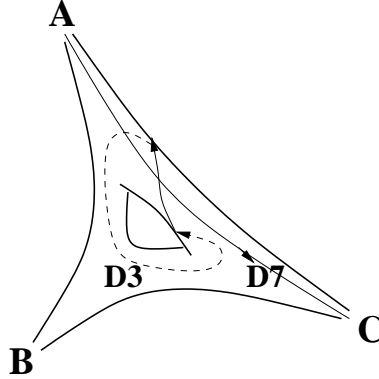


Figure 16: Intuitive picture of the 1-cycles in Σ associated with D3 and D7-branes.

The origin of the Chan-Paton multiplicity is clear in this description. As one can see in Figure 12, for each pair of punctures there are three possible paths that the associated D7-brane 1-cycle can take. These are explicitly shown in Figure 17. For fixed pair of punctures, the different 1-cycles correspond to different possible supersymmetric D7-branes wrapped on the corresponding 4-cycle. Namely, to different choices of Chan-Paton factors. Indeed, for a fixed pair of punctured the different 1-cycles are related by the \mathbb{Z}_3 geometric symmetry,

²⁰The complete 3-cycle, along with its supersymmetry, are discussed later on.

mirror to the \mathbb{Z}_3 quantum symmetry of the orbifold theory, in agreement with the Chan-Paton interpretation. To make contact with standard orbifold notation, 1-cycles of type BA, AC, CB correspond to D7-branes wrapped on the 4-cycles $z_i = 0$, for $i = 1, 2, 3$ respectively (often denoted $D7_1, D7_2, D7_3$ in the orbifold/orientifold literature). On the other hand, the choices a, b, c correspond to different choices of the Chan-Paton action.

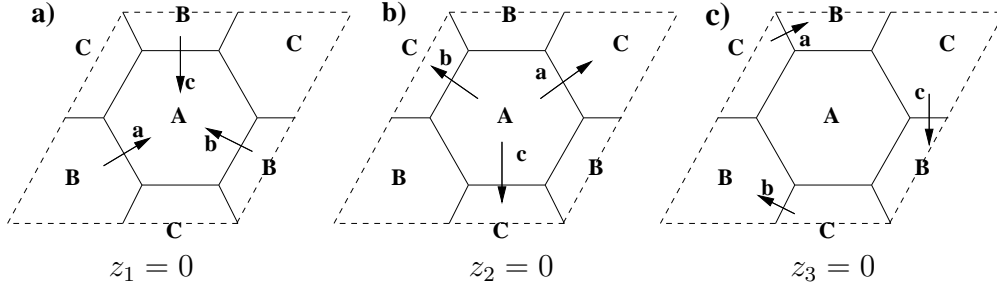


Figure 17: Different 1-cycles corresponding to the mirrors of D7-brane 4-cycles for the dP_0 theory. The segments represent 1-cycles stretched between the punctures located at the position of the face labels.

It is easy to obtain the 3-7 spectrum of chiral multiplets by simply computing the intersection numbers of the 1-cycles associated with the D7-brane of interest, and the 1-cycles associated with the gauge factors of the D3-branes. This gives rise to the extended quivers in Figure 18. This agrees with the spectra that one can find using techniques of D-branes at orbifolds and Chan-Paton factors.

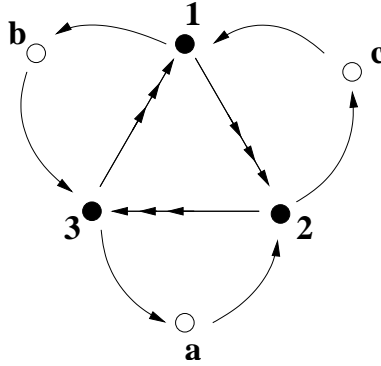


Figure 18: Extended quiver for the dP_0 theory with D7-branes.

Notice that different D7-branes on the same 4-cycle but with different choice of Chan-Paton factors lead to different D3-D7 spectrum. Also notice that different D7's can give rise

to the same spectrum of 3-7 chiral multiplets. However, different possibilities give rise to different superpotential interactions of type 33-37-73. These interactions are easily computed by considering disks bounded by two D3-branes and one D7-brane, shown in Figure 19. These are also in agreement with results from orbifold computations (they correspond to the interactions usually denoted $(33)_i$ -37-7 $_i$ 3, where $(33)_i$ denotes a bi-fundamental arising from the orbifold projection of the chiral multiplet parametrizing motion in the complex direction z_i in the parent $\mathcal{N} = 4$ theory of D3-branes in flat space).

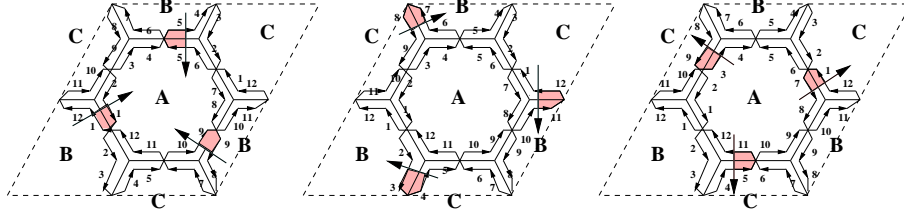


Figure 19: Disks corresponding to 33-37-73 interactions in the dP_0 theory.

Recall that each edge in the tiling of Σ corresponds to a bi-fundamental multiplet in the 33 sector. Hence our example illustrates that for each such bi-fundamental there is a possible supersymmetric D7-brane, given by the 1-cycle crossing the edge. Moreover, a little thought on the above pictures reveals that a bi-fundamental $\Phi_{ij} = (\square_i, \square_j)$ is associated with a D7-brane whose 73 and 37 sectors transform as \square_i and \square_j respectively, and that there is a 33-37-73 interaction between the three fields. These features follow from the dimer construction and are valid for general singularities.

Finally, in the presence of several D7-branes, there are in general non-trivial D7-D7' open string sectors. These are higher-dimensional fields, but are relevant for the 4d theory due the existence of 37-77'-7'3 couplings. Namely, vevs for 77' fields appear as mass terms for flavors of the D3-brane gauge theories.

The 77' fields and their interactions, can be determined using orbifold techniques. Concretely, two D7-branes on different 4-cycles $z_i = 0$, $z_j = 0$, (denoted D7 $_i$ -D7 $_j$) and with different Chan-Paton actions lead to one six-dimensional 7 $_i$ -7 $_j$ field (propagating on the extra $z_i = z_j = 0$ complex plane). The dimensionality of the D7-branes intersection is determined by the number of common punctures along which the corresponding non-compact 1-cycles go to infinity. At the origin, one of the 4d $\mathcal{N} = 1$ chiral multiplets in this 6d hypermultiplet has a superpotential coupling to the flavors in the D3-D7 $_i$ and D3-D7 $_j$ sectors. Also, two D7-branes on the same 4-cycle $z_i = 0$ (denoted D7 $_i$, D7 $_i$ ') but different choices of

Chan-Paton action lead to one eight-dimensional field. At the origin one 4d $\mathcal{N} = 1$ chiral multiplet couples to the flavors in the D3-D7_i and D3-D7_i' sectors. Finally, pairs of D7-branes with same choice of Chan-Paton action lead to higher-dimensional fields, but which do not couple to D3-D7 states (the orbifold projection forces these D7-brane fields to vanish at the origin).

In contrast with 33 and 37 states, 77' states are not perfectly characterized in the dimer diagram. This is related to the fact that they have non-compact support, hence they cannot be properly described in terms of intersections of 1-cycles (which lead to essentially four-dimensional fields). Heuristically, one could associate such fields to ‘intersections’ of D7-brane 1-cycles with common punctures. Using this picture, some $37_i - 7_i 7_j - 7_j 3$ interactions can be pictured in terms of disks as shown in Figure 20. However, notice that there are additional interactions that cannot be visualized in this way. For instance, the couplings $37_i - 7_i 7'_i - 7'_i 3$ exist, despite the fact that they do not correspond to disks in Σ (see Figure 21). Hence, interactions of 77' states with the 4d theory cannot be directly read out from the dimer diagram.

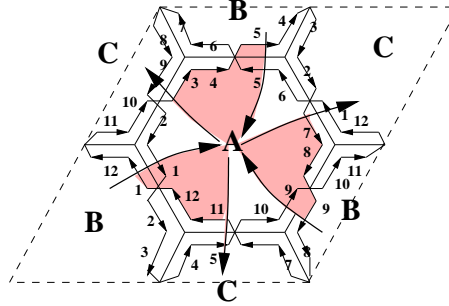


Figure 20: Examples of interactions $37_i - 7_i 7_j - 7_j 3$ in the dP_0 theory.

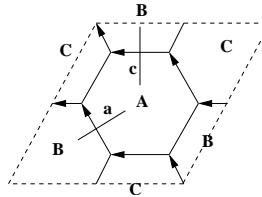


Figure 21: The non-trivial interaction $37_i - 7_i 7'_i - 7'_i 3$, which can be shown to exist using orbifold techniques, is not manifest as a disk in the mirror Riemann surface.

As mentioned, vevs for $77'$ states correspond to mass terms for certain D3-D7 open strings. Sketchily, the string interpretation of this fact is that the 4-cycles associated with the D7 and the $D7'$ recombine into a 4-cycle which does not pass through the singular point, but at some distance from it. A prototypical example (e.g. in \mathbb{C}^3) is to recombine D7-branes along $z_1 = 0$ and along $z_2 = 0$ (i.e. $z_1 z_2 = 0$) to D7-branes along $z_1 z_2 = \epsilon$. This gives a non-trivial mass to the 37 and $7'3$ open strings, which have a minimal non-zero stretching related to ϵ .

As a final piece of information, we would like to provide a more detailed description of the 3-cycles in the mirror geometry corresponding to the D7-branes, and to discuss their supersymmetry properties. Recall that the mirror geometry is given as a $\Sigma \times \mathbb{C}^*$ fibration over a complex plane \mathbb{C} . Let z be a (uniform) complex coordinate in our genus 1 Riemann surface, and let the \mathbb{C}^* fibration be described by $xy = w$, where w is the complex coordinate on the \mathbb{C} base. The holomorphic 3-form of the geometry is

$$\Omega = dw dz \frac{dx}{x} \quad (\text{B.1})$$

D3-branes are mirror to D6-branes on 3-cycles which span the \mathbf{S}^1 direction in \mathbb{C}^* (given by the orbit of $x \rightarrow e^{it}x$, $y \rightarrow e^{-it}y$), times a segment in \mathbb{C} (locally of the form $w = e^{i\theta}r$), times a 1-cycle in Σ (locally of the form $z = e^{-i\theta}s$. Here r, s, t are local coordinates on the 3-cycle. The 3-cycle is supersymmetric since it is special lagrangian with respect to $e^{-i\pi/2}\Omega$ (namely they are calibrated by $\text{Im } \Omega$). Indeed

$$\Omega|_{D3} = i dr ds dt \quad (\text{B.2})$$

So $\text{Re } \Omega|_{D3} = 0$, $\text{Im } \Omega|_{D3} = d\text{vol}_3$.

The 3-cycles mirror to D7-branes correspond to 1-cycles parallel to some D3-brane 1-cycle in Σ (compare Figures 14 and 17), hence along $z = e^{-i\theta}$ (note however that they stretch between punctures, so they are non-compact). In addition, we need to specify the two additional directions. They span a semi-infinite line in \mathbb{C} described by $w = e^{i\theta}s$ for $s \geq 0$, and the \mathbf{S}^1 direction in \mathbb{C}^* . Such 3-cycles are non-compact and calibrated by $\text{Im } \Omega$, hence preserve the same supersymmetry as the D3-brane 3-cycles ²¹.

A last important point is that for the above 3-cycles, the intersection numbers of the different 3-cycles is simply given by the intersection numbers of the 1-cycles in Σ .

²¹Clearly, there are other non-compact and supersymmetric 3-cycles in the geometry (mirror to other B-type branes, like D9-branes with holomorphic gauge bundles). The identification of the above ones as mirror of the D7-branes is ensured by the fact that the intersection numbers with the D3-brane 3-cycles reproduce the same D3-D7 spectrum as with orbifold techniques.

B.2 Generalization

The above story admits a natural generalization to any toric singularity. The general lesson is that D7-branes wrapped on holomorphic 4-cycles correspond to 1-cycles stretching between two punctures in the Riemann surface ²². More specifically, for each bi-fundamental field in the D3-D3 sector, one can construct a supersymmetric D7-brane, with a superpotential coupling 33-37-73 to precisely such bi-fundamental ²³.

As discussed above, this rule is manifest in the dimer graph. Moreover it implies that for a fixed choice of a pair of punctures, i.e. for a fixed 4-cycle, there may be a multiplicity of different D7-branes, which differ by the choice of ‘Chan-Paton action’. For non-orbifold singularities, this requires some explaining. Even in non-orbifold examples, D7-branes carry world-volume gauge degrees of freedom. Topologically different choices correspond to different D7-branes, in the sense that their D3-D7 spectrum and interactions are different. For instance, the choice of Chan-Paton factors in an orbifold model corresponds to such a topological choice, given by the holonomy of the gauge connection at infinity. This notion is however general, and can be used even in non-orbifold examples. Namely, the region at infinity in a non-compact 4-cycle is given, for toric geometries, by a Lens space $\mathbf{S}^3/\mathbb{Z}_n$. The value of n for a given 4-cycle can be obtained from the web diagram (equivalently from the 1-cycle in Σ) by computing the bilinear form $n = p_1 q_2 - q_1 p_2$ for the (p_i, q_i) charges of the legs/punctures associated with it. Since $H_1(\mathbf{S}^3/\mathbb{Z}_n) = \mathbb{Z}_n$, the holonomy of the gauge connection at infinity is characterized by an element of \mathbb{Z}_n . This implies that there are n possible choices of asymptotic behavior of the Chan-Paton bundle corresponding to D7-branes on such 4-cycle. Effectively, this corresponds to n different choices of D7-brane, in the sense that each choice leads to a different D3-D7 spectrum and interactions.

In fact this is in remarkable agreement with the dimer diagram picture, where there are indeed n different ways to connect the two punctures by crossing an edge in the tiling of Σ .

²² This relation between bi-fundamentals and 4-cycles is isomorphic to another familiar relation, see e.g.[48]: given a bi-fundamental multiplet, one can consider the corresponding dibaryonic operator. In the dual $\text{AdS}_5 \times Y_5$, where Y_5 is the base of the conical singularity X_6 , the dual to the dibaryon is a D3-brane wrapped on a supersymmetric 3-cycle C_3 in X_5 . The cone over C_3 is a holomorphic 4-cycle in X_6 . This is the 4-cycle which we are using, in a different setup, to wrap our D7-branes. Notice that the discussion below on Chan-Paton factors is isomorphic to that in section 2.2.1 of [48]. Also note that this correspondence between bi-fundamentals and D7-branes is valid for a general singularity, even non-toric ones.

²³It is important to clarify that in general all these D7-branes are not independent. In the language of footnote 22, the dibaryons of the bi-fundamental fields are not all independent. Rather, we use this rule to generate a (possibly redundant) class of D7-branes, their D3-D7 spectra and their interactions, in a simple fashion.

Given a D7-brane, the computation of the D7-D3 spectrum is given by the intersection numbers of the corresponding 1-cycles, as described above. Similarly, the 37-73-33 interactions correspond to disk diagrams in Σ , and lead to a coupling of the D3-D7 branes to the 33 bi-fundamental associated to the D7-brane.

Finally, the 77' sector and its interactions with D3-D7 states are not properly encoded in the dimer diagram. We leave the general question of characterizing this sector for non-orbifold singularities as an open question. In the next section, the results we use for the dP_1 theory are obtained by requiring consistency upon higgsing the dP_1 to the dP_0 theory. For a general toric singularity, the interactions between 77' and D3-D7 states can be determined by computing them in a sufficiently large abelian orbifold and partially resolving it to obtain the singularity of interest.

B.3 D7-branes for the dP_1 theory

Let us now consider the case of interest in the main text, namely the dP_1 theory. The unit cell of the dimer diagram and the zig-zag paths are shown in Figure 22a. The web diagram is shown in Figure 22b.

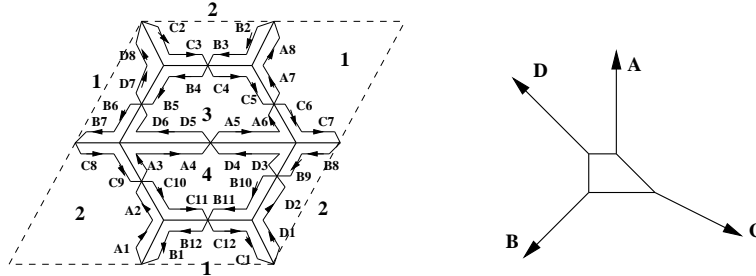


Figure 22: (a) Dimer diagram and zig-zag paths in the dP_1 theory. (b) Web diagram.

The mirror Riemann surface is shown in Figure 23.

Notice the close relation with the dP_0 theory, which amounts (up to a trivial relabeling) to the removal of the edge separating faces 3 and 4 in the dimer diagram, and of the edge separating faces A and D in the tiling of Σ .

The 1-cycles in Σ corresponding to the different D3-brane gauge group factors are shown in Figure 24

Their intersection number reproduces the quiver of the dP_1 theory, see Figure 25

Now we can construct the D7-branes associated with the different bi-fundamentals, and obtain the 3-7 spectrum by computing the intersection numbers. The different D7-branes,

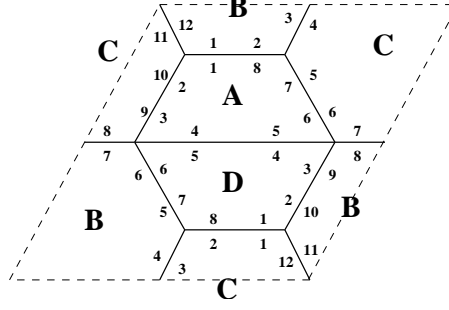


Figure 23: Mirror Riemann surface for the dP_1 theory. The fact that it comes out to be similar to the original brane tiling is a property of del Pezzo theories, and not valid for a general singularity.

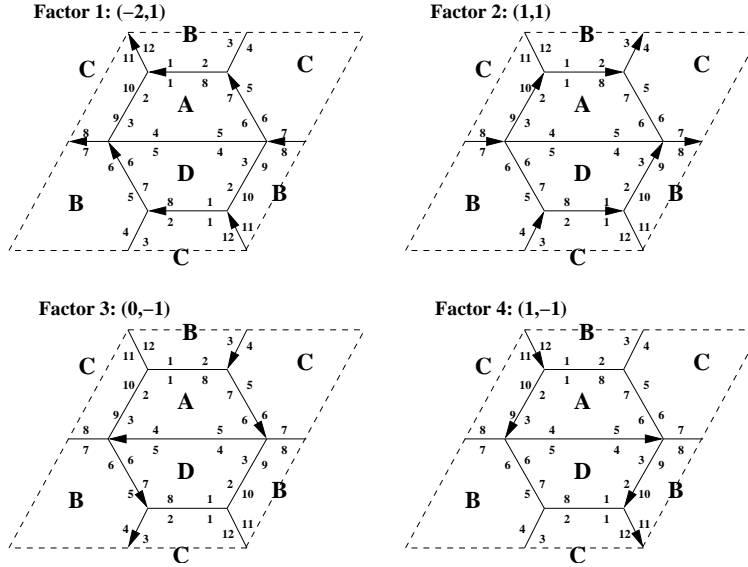


Figure 24: The 1-cycles for the four D3-brane gauge factors in the dP_1 theory.

and the resulting extended quivers are shown in Figure 26. It is a straightforward exercise to write down the explicit interaction terms in the presence of these objects (they correspond to oriented triangles in the extended quivers). In Figure 26 we have indicated the 33 bifundamental which couples to the 37 and 73 states for each choice of D7-brane.

In the main text we consider introducing a set of D7-branes corresponding to Σ_{AB} , Σ_{BC} and Σ_{DB} . In order to determine the possible mass terms for the D3-brane flavors, we need to obtain the coupling of $77'$ states to states in the 37 and $37'$ sectors, for the different

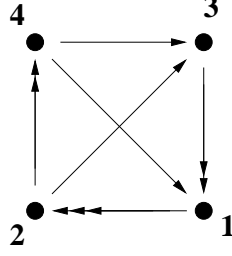


Figure 25: Quiver diagram for the dP_1 theory.

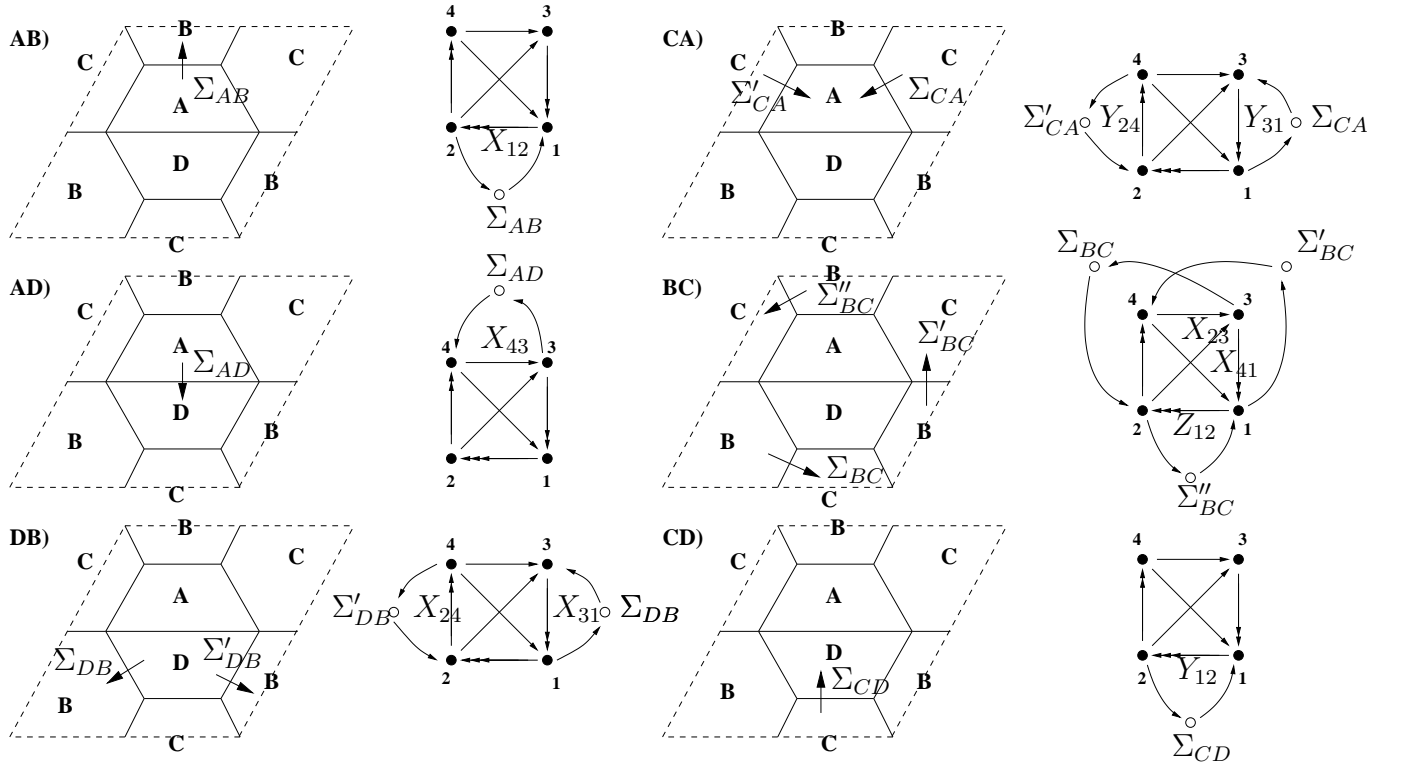


Figure 26: The 1-cycles and extended quivers for different kinds of D7-branes in the dP_1 theory. In the quiver we have indicated the 33 bi-fundamental which couples to the corresponding 37 and 73 states.

choices of D3-brane gauge factors. In Figure 27 we show the 1-cycles for the gauge factors 1, 2 and 3 (namely those with extra flavors), along with the D7-brane 1-cycles intersecting them. Figures (b) and (c) lead to disk diagrams of the kind in Figure 20, making manifest the existence of interactions leading to flavor masses. Figure (a) does not contain disks, but is the analog of Figure 21. Namely there is a $37 - 77' - 7'3$ coupling leading to masses for

flavors of gauge factor 1. This can be shown by requiring consistency with the existence of such coupling in the dP_0 theory, upon higgssing the dP_1 theory.

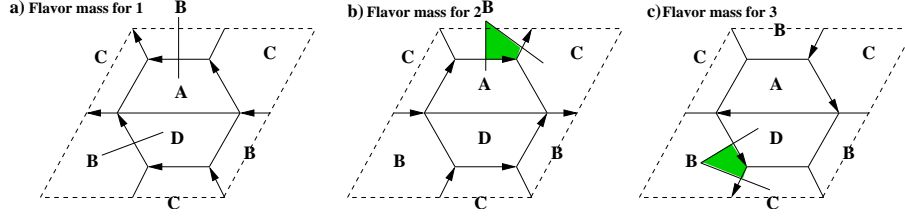


Figure 27: D3 and D7-branes in the dP_1 theory studied in Section 4. For each D3-brane gauge factor we have shown the D7-branes leading to the corresponding flavors.

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